MAE 104 - SUMMER 2015
HOMEWORK 1 SOLUTION

Problem 1:

1. Plot the drag coefficient, c_D, the aerodynamic efficiency, AE, and the center of pressure, x_{cp}, of the airplane as a function of the angle of attack.

   By combining together the plots for the polar curve and the c_L as a function of α, we obtain the c_D data. The result is shown in Figure 1.

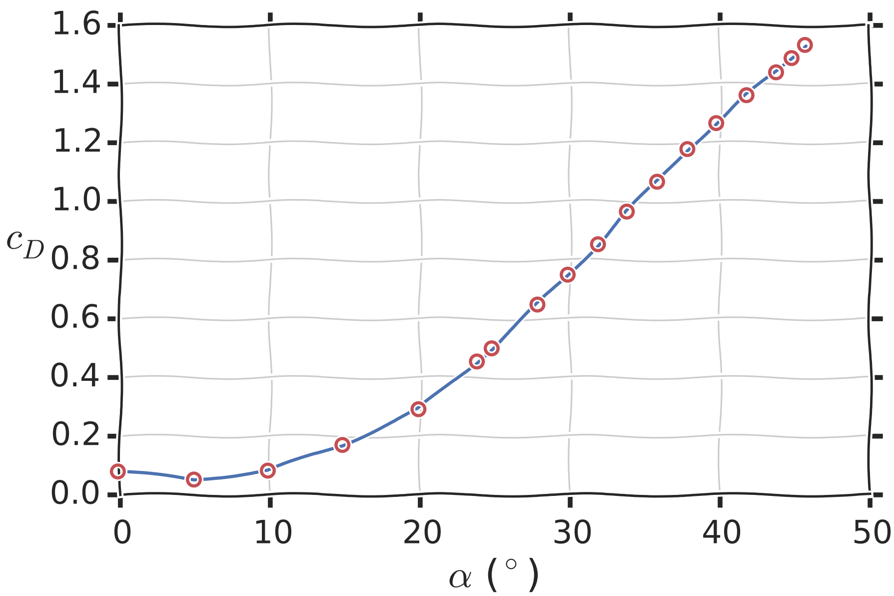

Figure 1: Drag coefficient of the airplane as a function of the angle of attack.

The aerodynamic efficiency is defined as $AE = c_L/c_D$. By combining the c_L and c_D plots we get AE as a function of α, represented in Figure 2.

Consider the sketch presented in Figure 3 for the geometry of the problem. The origin of the (x, y) axis is located at the front of the airplane.

Applying the definition of center of pressure:

$$M_{cp} = N \cdot \left(x_{cp} - x_{c/4}\right) + M_{c/4} = 0.$$

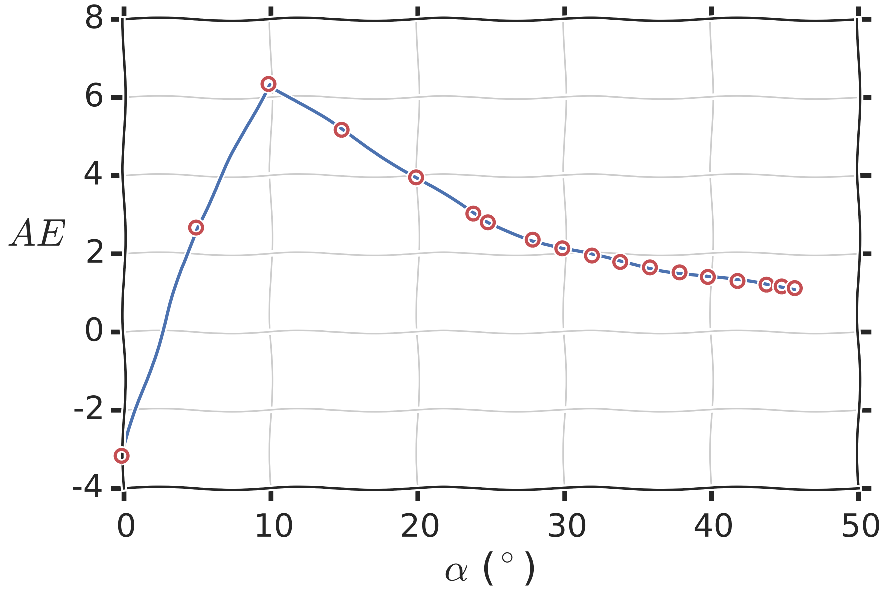

Figure 2: Aerodynamic efficiency of the airplane as a function of the angle of attack.

By using the definition of the aerodynamic coefficients, defining the average chord length as $\bar{c} = S/b$, and using the relation

$$c_N = c_L \cdot \cos(\alpha) + c_D \cdot \sin(\alpha),$$

we find that the location of the center of pressure is

$$\boxed{x_{cp} = 0.6 \cdot d - \frac{S}{b} \cdot \frac{c_{M_{c/4}}}{c_N},}$$

Figure 4 represents the location of the center of pressure as a function of the angle of attack.

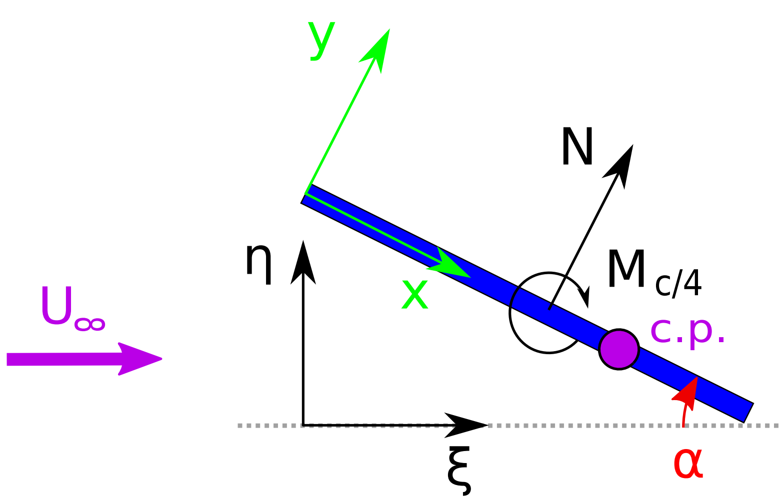

Figure 3: Sketch for the geometry of the problem.

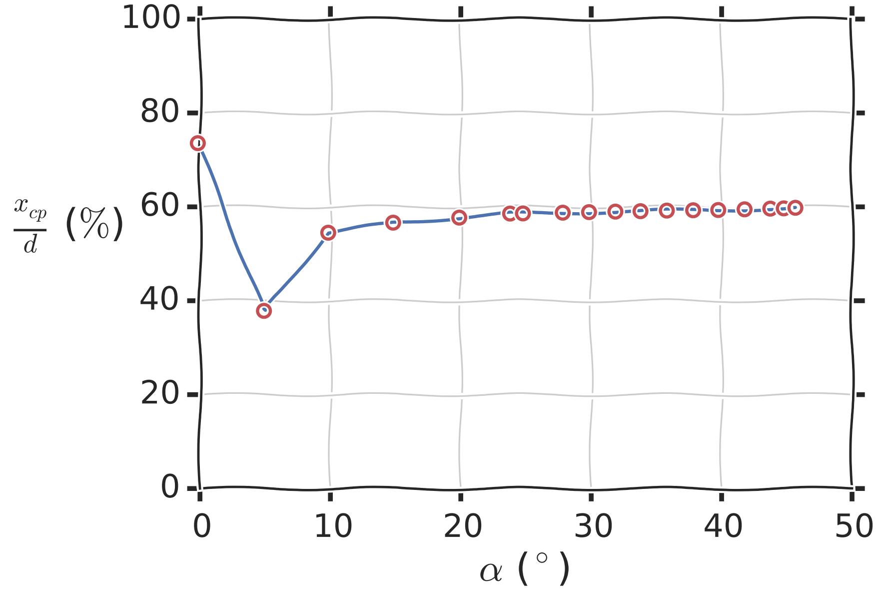

Figure 4: Center of pressure as a function of the angle of attack.

2. Calculate the stalling velocity of the airplane. What are α, c_D, $c_{M_{c/4}}$ and AE at this flight condition?

$$c_{L,max} \approx 1.8 \Rightarrow \boxed{V_{stall} = \sqrt{\frac{2W}{\rho S c_{L,max}}} \approx 74.2 \text{ m/s},}$$

$$\boxed{\alpha_{stall} \approx 38^\circ,} **$$

$$\boxed{c_{D,stall} \approx 1.2,}$$

$$\boxed{c_{M_{c/4},stall} \approx 0.09,}$$

$$\boxed{AE_{stall} \approx 1.5.}$$

** The angle of attack α_{stall} is very large. Since the engines are fixed to the airplane, the thrust at high angles of attack have a non-zero component parallel to the lift. Now that we know that α is large, we need to take into account this component of the thrust in order to calculate the actual V_{stall}. However, this is out of the scope of this problem, and for simplicity, here on I will use the approximation of small α (all the text in blue), and I will consider it as good in my grading. If you are interested in knowing the actual solution, keep reading. Consider the force balance plotted in Figure 5. Equilibrium of forces renders

$$W = L + T \cdot \sin(\alpha),$$
$$D = T \cdot \cos(\alpha),$$

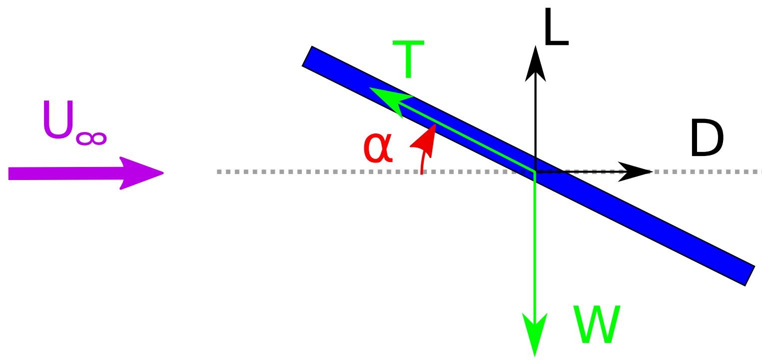

Figure 5: Force balance for high angles of attack.

and thus:

$$\frac{1}{2}\rho V^2 S \cdot [c_L + c_D \tan(\alpha)] = W,$$

$$V_{stall} = \sqrt{\frac{2W}{\rho S \left[c_L + c_D \cdot \tan(\alpha)\right]_{max}}}$$

The stall doesn't happen now at the maximum c_L but at the maximum $c_L + c_D \cdot \tan(\alpha)$, which is the coefficient of total force counteracting the airplane's weight. Figure 6 depicts the dependence of $c_L + c_D \cdot \tan(\alpha)$ with α. It can be seen that this coefficient of force doesn't reach a maximum for the range of angles of attack provided. This means that the airplane can reach very high angles of attack and thus very low speeds without stalling. Potentially, if the curve doesn't reach a maximum, in a flight with lower weight, the airplane could stay vertically with zero speed and balancing the

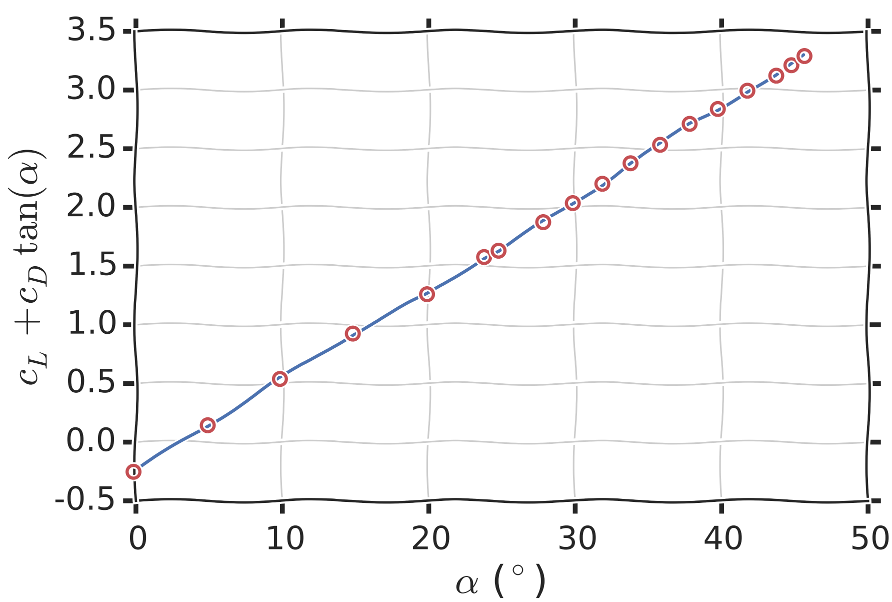

Figure 6: Coefficient of total force counteracting the airplane's weight.

Figure 7: Airplane flying at $\alpha = \dfrac{\pi}{2}$ rad.

weight with just the thrust provided by the engines. And just to finish, Figure 7 shows a cool picture of a MIG-29 performing the cobra maneuver in which it stays vertical, with zero velocity, and balanced just by the engines' thrust

3. Under the conditions described in the previous part, calculate the lift, drag, moment **with respect to the front of the airplane** M_0 and traction acting on the airplane.

$$\boxed{L_{stall} = W = 230,535 \text{ N},}$$

$$\boxed{D_{stall} = \frac{1}{2}\rho V_{stall}^2 S c_{D,stall} \approx 151,700 \text{ N},}$$

$$M_{0,stall} = -N \cdot 0.6 \cdot d + M_{c/4} = \frac{1}{2}\rho V_{stall}^2 S \bar{c}\left[-0.6\frac{d}{\bar{c}}\left(c_{L,max}\cos(\alpha) + c_{D,stall}\sin(\alpha)\right) + c_{M_{c/4}}\right]$$

$$\boxed{M_{0,stall} \approx -2,788,496 \text{ N} \cdot \text{m},}$$

$$\boxed{T_{stall} = D_{stall} \approx 151,700 \text{ N}.}$$

4. Calculate the maximum velocity of the airplane. What are α, c_L, $c_{M_{c/4}}$ and AE at this flight condition?

The airplane has two engines, and thus the maximum thrust is $T_{max} = 2T$.

$$c_{D,min} \approx 0.05 \Rightarrow \boxed{V_{max} = \sqrt{\frac{2T_{max}}{\rho S c_{D,min}}} \approx 360.6 \text{ m/s},}$$

$$\boxed{\alpha_{c_{D,min}} \approx 5^\circ,}$$

$$\boxed{c_{L,c_{D,min}} \approx 0.14,}$$

$$\boxed{c_{M_{c/4},c_{D,min}} \approx 0.17,}$$

$$\boxed{AE_{c_{D,min}} \approx 2.6.}$$

5. Under the conditions described in the previous part, calculate the lift, drag, moment **with respect to the front of the airplane** and traction acting on the airplane.

$$\boxed{L_{c_{D.min}} = \frac{1}{2}\rho V_{max}^2 S c_{L,c_{D,min}} = 413,596 \text{ N} \neq W,}$$

$$\boxed{D_{c_{D,min}} = T_{max} = 158,400 \text{ N},}$$

$$M_{0,c_{D,max}} = -N \cdot 0.6 \cdot d + M_{c/4} = \frac{1}{2}\rho V_{max}^2 S\bar{c}\left[-0.6\frac{d}{\bar{c}}\left(c_{L,c_{D,min}}\cos(\alpha) + c_{D,min}\sin(\alpha)\right) + c_{M_{c/4}}\right]$$

$$\boxed{M_{0,c_{D,min}} \approx -2,754,348 \text{ N}\cdot\text{m},}$$

$$\boxed{T_{c_{D,min}} = T_{max} = 158,400 \text{ N}.}$$

6. What is the angle of attack that maximizes AE? Calculate the lift to drag ratio, velocity, lift L, drag D, moment **with respect to the front of the airplane** M_0 and thrust T for that angle of attack.

$$\boxed{\alpha_{AE_{max}} \approx 10^\circ,}$$

$$\boxed{AE_{max} \approx 6.3,}$$

$$\boxed{V_{AE_{max}} = \sqrt{\frac{2W}{\rho S c_{L,AE_{max}}}} \approx 137.4 \text{ m/s},}$$

$$\boxed{L_{AE_{max}} = W = 230,535 \text{ N},}$$

$$\boxed{D_{AE_{max}} = \frac{W}{AE_{max}} \approx 36,332 \text{ N},}$$

$$M_{0,AE_{max}} = -N \cdot 0.6 \cdot d + M_{c/4} = -0.6 \cdot d \cdot (L_{AE_{max}}\cos(\alpha) + D_{AE_{max}}\sin(\alpha)) + \frac{1}{2}\rho V_{AE_{max}}^2 S\bar{c}c_{M_{c/4}}$$

$$\boxed{M_{0,AE_{max}} \approx -2,174,243 \text{ N}\cdot\text{m},}$$

$$\boxed{T_{AE_{max}} = D_{AE_{max}} \approx 36,332 \text{ N}.}$$

7. When the airplane is at a height $h = 4,000m$. both engines are turned off ($T = 0N$.) and the airplane is glided to the ground. Calculate the maximum distance that the airplane is able to glide at maximum lift to drag ratio.

   If x is the traveled distance:

$$AE = \frac{x}{h} = \frac{1}{\tan(\alpha)} \Rightarrow \boxed{x_{AE_{max}} = AE_{max} \cdot h \approx 25,381 \text{ m.}}$$

8. Specify the units of all the previous results.

   All the results in the previous questions have the correct units.

Bonus: Is there anything wrong with questions 4. and 5.?

The velocity is supersonic, and thus the theory used doesn't hold.

Problem 2:

1. What is the relation between the lift L' and normal force N' on the airfoil? Justify why we can integrate along the reference system (x_2, y_2) instead of (x, y) to calculate L'.

   The integral of c_p on the surface of the airfoil, projected on (x_2, y_2) will render the normal and axial forces. However, we are interested in the lift force

$$L' = N' \cdot \cos(\alpha) - A' \cdot \sin(\alpha)$$

   Given that α is very small:

$$\boxed{L' \approx N'.}$$

2. Integrate c_p along the surface of the airfoil to calculate L'. **Hint:** use the coordinate system (x_2, y_2).

$$c_l \approx c_n = \frac{1}{c} \cdot \left\{ - \int_{x_2=-c/2}^{x_2=c/2} c_{p,u}(x_2) dx_2 + \int_{x_2=-c/2}^{x_2=c/2} c_{p,l}(x_2) dx_2 \right\} =$$

$$\frac{1}{c} \cdot \int_{x_2=-c/2}^{x_2=c/2} \left\{ 2\alpha \sqrt{\frac{1 - 2x_2/c}{1 + 2x_2/c}} + \alpha^2 \frac{1 - 2x_2/c}{1 + 2x_2/c} + 2\alpha \sqrt{\frac{1 - 2x_2/c}{1 + 2x_2/c}} - \alpha^2 \frac{1 - 2x_2/c}{1 + 2x_2/c} \right\} dx_2 =$$

$$\frac{1}{c} \cdot \int_{x_2=-c/2}^{x_2=c/2} 4\alpha \sqrt{\frac{1 - 2x_2/c}{1 + 2x_2/c}} dx_2$$

   Making the change of variable $x_2 = \frac{c}{2}\xi$:

$$c_l \approx 2\alpha \cdot \int_{\xi=-1}^{\xi=1} \sqrt{\frac{1-\xi}{1+\xi}} d\xi = 2\pi\alpha$$

$$\boxed{L' = \frac{1}{2}\rho U_\infty^2 S c_l \approx \rho U_\infty^2 S \pi \alpha}$$

3. Calculate the coefficient of moment at the leading edge of the airfoil, $c_{m_{L.E.}}$.

Using the reference system (x_2, y_2):

$$c_{m,L.E.} = \frac{1}{c^2} \cdot \int\limits_{x_2=-c/2}^{x_2=c/2} \Big[c_{p,u}(x_2) - c_{p,l}(x_2) \Big] \left(x_2 + \frac{c}{2} \right) dx_2 =$$

$$\frac{1}{c^2} \cdot \int\limits_{x_2=-c/2}^{x_2=c/2} -4\alpha \sqrt{\frac{1 - 2x_2/c}{1 + 2x_2/c}} \left(x_2 + \frac{c}{2} \right) dx_2$$

Making the change of variable $x_2 = \frac{c}{2}\xi$:

$$\boxed{c_{m,L.E.} = -\alpha \cdot \int\limits_{\xi=-1}^{\xi=1} \sqrt{\frac{1-\xi}{1+\xi}} \, (\xi + 1) \, d\xi = -\frac{\alpha}{2}\pi}$$

4. Calculate the coefficient of moment at the $c/4$ point, $c_{m_{c/4}}$.

$$c_{m,c/4} = c_{m,L.E.} + \frac{1}{4} c_l$$

$$\boxed{c_{m,c/4} = 0}$$