MAE 104 - SUMMER 2015
HOMEWORK 2

Due Tuesday 08-18-2015 in class

Guidelines:
Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required to understand your solution. Required plots should be generated using computer software such as Matplotlib or LibreOffice. Remember to specify all the units of your results.

Problem 1:
Two counter-rotating vortices are located at $(x, y) = (0, 0)$ and $(x, y) = (a, 0)$, respectively, and subjected to a uniform flow U_∞, as shown in Figure 1.

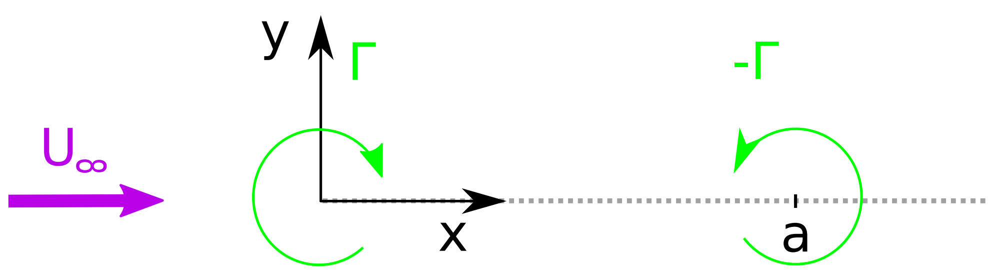

Figure 1: Singularity distribution.

1. Find the stream function ψ of the flow by adding the contributions of the different elementary flows that constitute it.
2. Calculate the vorticity $\vec{\omega} = \nabla \wedge \vec{v}$. With this result in hand, does the potential function ϕ exist? If so, calculate it by applying the compatibility conditions.
3. Calculate the velocity field in both Cartesian and polar coordinates. What is the velocity far downstream?
4. Calculate the stagnation points of the flow. Try solving first the equation $v = 0$.
Note: the stagnation points are the points with zero velocity.

5. Sketch the flow around the singularities, clearly indicating the dividing streamlines. **Note:** the dividing streamlines are the streamlines that pass through the stagnation points. They are the only place on the flow, apart from the singularities, where streamlines can cross.

6. Calculate the lift generated by the total flow.

7. Choose any closed path that encloses both singularities and calculate the circulation around it. Choose any path that encloses just one of the singularities and calculate the circulation around it. **Note:** Perform all the integrals.

Problem 2:
An airfoil is subjected to a steady free stream and it generates a viscous wake, as seen in Figure 2.

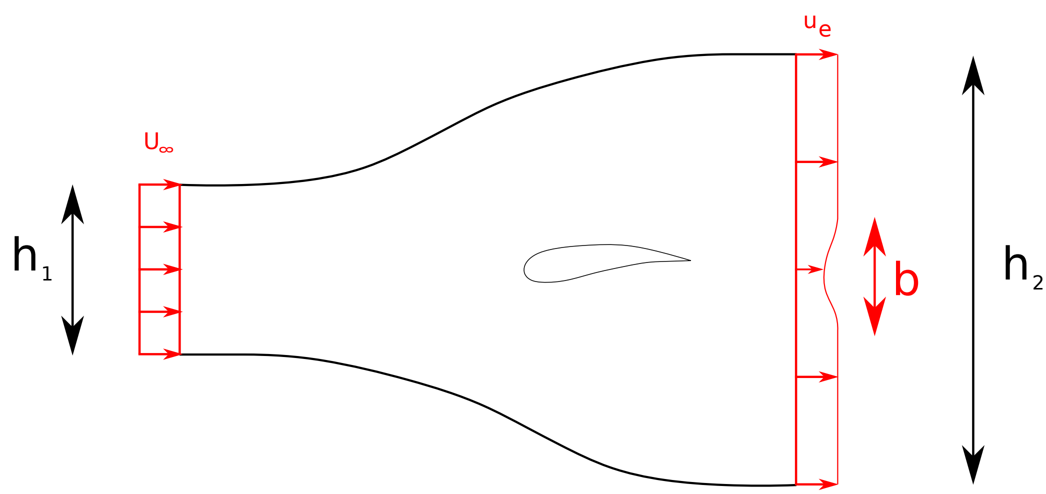

Figure 2: Sketch of an airfoil being tested in a wind tunnel.

In the experiment, we measure an upstream velocity U_∞ and a downstream velocity distribution $u_e = U_\infty - u_d$ such that:

$$u_d(y) = \begin{cases} A \cdot \left(1 + \cos\left(\frac{2\pi y}{b}\right)\right) & ; \quad |y| \leq \frac{b}{2} \\ 0 & ; \quad |y| \geq \frac{b}{2} \end{cases}$$

where b is the wake thickness. Both upstream and downstream pressure are constant of value p_∞. The pressure far away, both above and below it, is also p_∞.

1. Choose an appropriate control volume to apply the conservation equations.

2. Apply the integral form of the continuity equation and obtain the wake thickness b as a function of the rest of the problem's parameters.

3. Apply the integral form of the momentum equation and obtain the drag force D' as a function of the rest of the problem's parameters.