MAE 104 - SUMMER 2015
HOMEWORK 3

Due Tuesday 08-25-2015 in class

Guidelines:
Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required to understand your solution. Required plots should be generated using computer software such as Matplotlib or LibreOffice. Remember to specify all the units of your results.

Problem 1:
A thin flat plate is flying with velocity V_∞ and angle of attack α, as shown in Figure 1. When α is small, the components of the velocity parallel and perpendicular to the plate are, for a region near the plate, $y << c$:

$$u(x,y) = V_\infty,$$
$$v(x,y) = V_\infty \alpha + w,$$

where w is the self induced velocity. The velocity field can be calculated by using *Thin Airfoil Theory*.

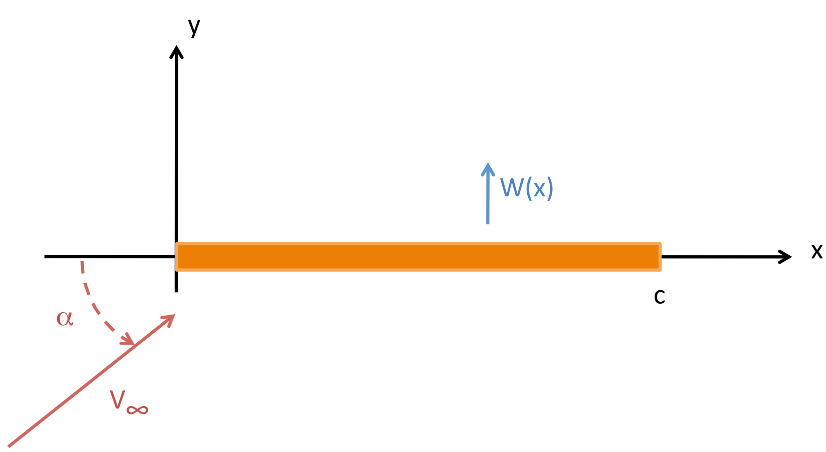

Figure 1: Thin plate.

1. First, substitute the thin plate by a vortex sheet such that the chord is a streamline. Calculate the intensity of the vortex sheet, $\gamma(\theta)$. Plot $\gamma(x)$.

2. Using the previously calculated $\gamma(\theta)$, calculate the vertical self-induced velocity w. Note that there are three flow regions with distinct vertical velocity: upstream, the plate surface and downstream.

3. Calculate the lift coefficient c_l of the plate. Include a sketch of the integration domain, a differential section and the differential of force that it generates. Compare your result with the solution of Homework 1 - Problem 2.

4. Calculate the moment coefficient about the leading edge of the plate and the moment coefficient about the quarter chord point. Include a sketch of the integration domain, a differential section and the differential of force and moment that it generates. Compare your results with the solution of Homework 1 - Problem 2.

5. The flat plate has a chord $c = 5$ m. It is flying with velocity $V_\infty = 60$ m/s through air of density $\rho = 1.23$ kg/m^3, and it generates a lift per unit span $L' = 7$ kN/m. Calculate its angle of attack for this flight condition. Remember to indicate the units of your result.

6. The flat plate is now flying with the same velocity and different angle of attack. It feels a moment (per unit span) around the leading edge $M'_{L.E.} = 5$ kN. Calculate the new angle of attack. Remember to indicate the units of your result.

7. Plot c_l, $c_{m,L.E.}$ and $c_{m,c/4}$ as a function of α.

Problem 2:

An airfoil is provided with a flap, as shown in Figure 2. It flies with velocity U_∞, and its front part is aligned with the incident flow. The flap deflects a small angle δ rad. We want to study the effect of the flap deflection by using *Thin Airfoil Theory*.

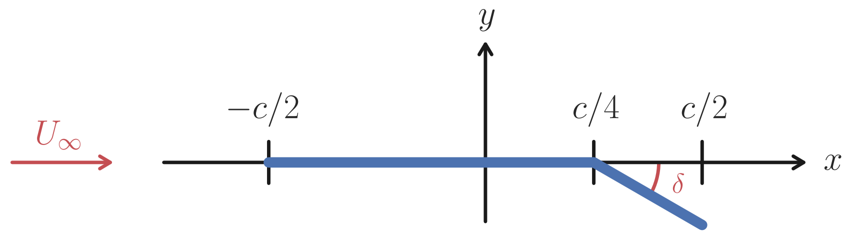

Figure 2: Airfoil with a deflected flap.

1. Draw a sketch of the camber line and the chord line. Include the free stream velocity at a non-horizontal configuration and clearly indicate α.

2. Find the slope of the camber line $dy_c(x)/dx$.
   Note 1: the equation of the camber line is defined as a distance to the chord line.
   Note 2: use the simplification of small angles.

3. Express the slope of the camber line as a Fourier series

$$\frac{dy_c(\theta_0)}{dx} = (\alpha - A_0) + \sum_{n=1}^{n=\infty} A_n \cdot \cos(n\theta_0),$$

   and calculate the coefficients A_0, A_1 and A_2 as a function of α and δ.

4. Substitute the thin plate by a vortex sheet such that the chord is a streamline. Calculate the intensity of the vortex sheet, $\gamma(\theta)$, as a function of the coefficients of the Fourier series A_n.

5. Calculate the circulation around a closed path enclosing the airfoil.

6. Calculate the lift L', lift coefficient c_l, lift slope a_0, angle of zero lift $\alpha_{L'=0}$ and the ideal angle of attack α_I of the airfoil. **Note:** the ideal angle of attack is defined as the angle at which $A_0 = 0$.

7. Calculate the coefficient of moment about the leading edge $c_{m,L.E.}$, the coefficient of moment about the quarter-chord point $c_{m,C/4}$ and the location of the center of pressure as a distance to the leading edge. Include a sketch of the moment balance used to calculate the location of the center of pressure.

8. Plot c_l, $c_{m,L.E.}$ and $c_{m,c/4}$ as a function of α for different values of δ.