MAE 104 - SUMMER 2015
HOMEWORK 4

Due Tuesday 09-01-2015 in class

Guidelines:
Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required to understand your solution. Required plots should be generated using computer software such as Matplotlib or LibreOffice. Remember to specify all the units of your results.

Problem 1:
Consider a wing with elliptical planform shape, aspect ratio $\Lambda = 10$, and formed by equal thin airfoils (with $\partial c_l/\partial\alpha = 2\pi$). Its non-dimensional circulation distribution is

$$G_b(\theta) = \frac{\Gamma}{bu_\infty} = -\frac{2\varepsilon}{\pi\Lambda} \cdot [3\sin(\theta) + \sin(3\theta)]\,,$$

where $\dfrac{y}{b} = \dfrac{1}{2}\cos(\theta)$.

1. Calculate the lift coefficient, c_L, of the wing when the unperturbed free-stream is parallel to the zero-lift direction ($\alpha_{l=0}$) of the central airfoil.
2. Calculate the induced drag coefficient of the wing, $c_{D,i}$.
3. Calculate the lift coefficient of each wing section as a function of ε and θ.
4. In the light of the result from the previous part, determine the airfoil in which the stall begins.

Problem 2:
Figure 1 shows a sketch of a wing of large aspect ratio $\Lambda \gg 1$.

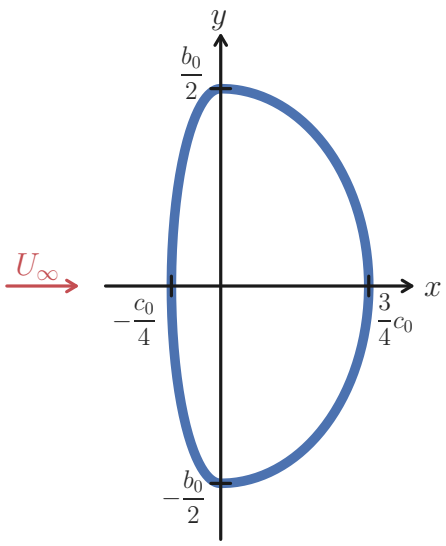

Figure 1: Sketch of the wing.

In the given axes, the expressions for the leading edge $x_{le}(y)$ and the trailing edge $x_{te}(y)$ of the wing are:

$$\frac{x_{le}(y)}{c_0} = -\frac{1}{4}\sqrt{1-\left(\frac{2y}{b_0}\right)^2},$$
$$\frac{x_{te}(y)}{c_0} = \frac{3}{4}\sqrt{1-\left(\frac{2y}{b_0}\right)^2}.$$

with $c_0 \ll b_0$.

We know that a generic airfoil of the wing ($-b_0/2 \leq y \leq b_0/2$) is a straight line that forms an angle

$$\varepsilon(y) = \varepsilon_0 \frac{|y|}{b_0},$$

with the root chord, where $\varepsilon_0 \ll 1$. We also know that the root chord forms an angle α with the unperturbed free-stream. If the unperturbed free-stream velocity is U_∞ and ρ the air density:

1. Write the form for the non-dimensional chord $K(\theta_0) = \dfrac{c(\theta_0)}{b_0}$ as a function of the wing's aspect ratio.

2. Write the form of the angle $\alpha(\theta_0) - \alpha_{l=0}(\theta_0)$, as a function of the parameters α, ε_0 and θ_0. Draw a sketch of this angle, clearly including the freestream, the root airfoil and a generic airfoil at a location θ_0.

3. Write the Fundamental Equation of Lifting Line theory. Substitute in it the solutions from the previous questions, and simplify it so that it is only a function of α, $\varepsilon(\theta_0)$, Λ, θ_0 and the coefficients of the circulation A_n.

4. In the equation derived in the last part, only the coefficient A_n are unknown. Calculate the coefficients A_n.

5. Calculate the lift L of the wing and the lift coefficient c_L.

6. Obtain the induced drag D_i on the wing and the induced drag coefficient $c_{D,i}$.