MAE 104 - SUMMER 2015
Problem Session 2

08-13-2015

Problem 1:
Given the velocity field:

$$u = \frac{x}{1+t},$$
$$v = \frac{y}{1+2t},$$
$$w = 0.$$

1. Calculate the vorticity of the flow. Does a potential function exist? If so, calculate it.
2. Calculate the divergence of the velocity. Does a stream function exist? If so, calculate it.
3. Calculate the streamlines, trajectories and path lines.
4. Calculate the streaklines.

Problem 2:
Consider the flow shown in Figure 1 consisting of a source of intensity Q at $(x, y) = (0, a)$, a uniform horizontal flow U_∞ and a second source of equal intensity at $(x, y) = (0, -a)$. Sketch the flow streamlines by following these steps:

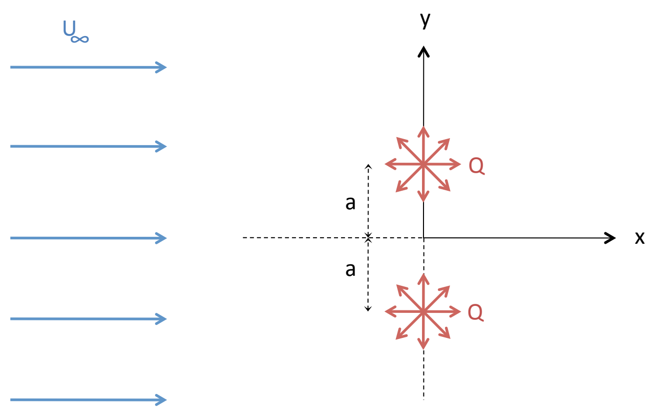

Figure 1: Two sources and an uniform flow.

1. Find the potential function ϕ of the flow by adding the contributions of the different elementary flows that constitute it.

2. Calculate, from the velocity potential function, the Cartesian velocity components (u, v) as a function of x, y, Q, U_∞ and a.

3. Apply the compatibility conditions to the potential function and calculate the stream function ψ.

4. Express u and v in polar coordinates by making the change of variables $x = r\cos\theta$, $y = r\sin\theta$, and non-dimensionalize them such that
$$\frac{u}{Q/2\pi a} = f(R, \theta, K),$$
$$\frac{v}{Q/2\pi a} = g(R, \theta, K),$$
where f and g are non-dimensional functions, $R = r/a$ and $K = 2\pi U_\infty a/Q$. This will simplify all your calculations in the following steps.

5. To find the stagnation points, solve first the equation $v = 0$. Obtain the two solutions.

6. Then use the result from part 5 on the equation $u = 0$. Obtain the stagnation points according to the values of K. Discriminate three different cases: $K = 1$, $K > 1$ and $K < 1$.

7. For each case ($K = 1$, $K > 1$ and $K < 1$), find the asymptotes of the stream function that passes through each stagnation point, for $x \to \pm\infty$. Use this information to sketch the streamlines.

Problem 3:
Consider the flow shown in Figure 2 consisting of a vortex of intensity Γ at $(x, y) = (0, a)$, a uniform horizontal flow U_∞ and a second vortex of equal intensity at $(x, y) = (0, -a)$.

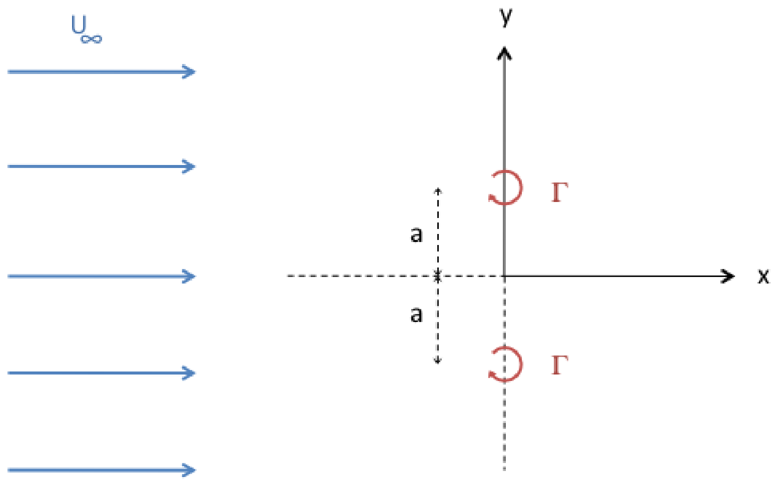

Figure 2: Two vortices and an uniform flow.

1. Find the stream function ψ of the flow by adding the contributions of the different elementary flows that constitute it.

2. Calculate, from the stream function, the Cartesian velocity components (u, v) as a function of x, y, Γ, U_∞ and a.

3. Apply the compatibility conditions to the stream function and calculate the potential function ϕ.

4. Express u and v in polar coordinates by making the change of variables $x = r\cos\theta$, $y = r\sin\theta$, and non-dimensionalize them such that
$$\frac{u}{\Gamma/2\pi a} = f(R, \theta, K),$$
$$\frac{v}{\Gamma/2\pi a} = g(R, \theta, K),$$
where f and g are non-dimensional functions, $R = r/a$ and $K = 2\pi U_\infty a/\Gamma$. This will simplify all your calculations in the following steps.

5. To find the stagnation points, solve first the equation $v = 0$.

6. Then use the result from part 5 on the equation $u = 0$. Obtain the 2 single stagnation points.

7. Find the asymptotes of the streamlines that pass through the stagnation points, for $x \to \pm\infty$. Use this information to sketch the streamlines.