MAE 104 - SUMMER 2015
Problem Session 4

08-27-2015

Problem 1:
Consider a wing with elliptical planform shape and large aspect ratio $\Lambda >> 1$, formed by equal airfoils with $\dfrac{\partial c_l}{\partial \alpha} = 2\pi$, flying with velocity U_∞. The unperturbed free-stream is parallel to the zero-lift direction ($\alpha_{l=0}$) of the central wing section. The non-dimensional circulation distribution on the wing has the form:

$$G_b(\theta) = \frac{\Gamma}{bU_\infty} = -\frac{7\delta}{5\pi\Lambda}\sin(\theta)$$

where $\dfrac{y}{b} = \dfrac{1}{2}\cos(\theta)$.

1. Write the equation of the chords as a function of y. Write it also as a function of θ.
2. Calculate the area of the planform, the aspect ratio and the non-dimensional chord.
3. Calculate the angle of attack of each wing section with respect to the zero-lift line of the section. Calculate also the twist of the wing.
4. Calculate the lift coefficient, c_L, the induced drag coefficient, c_{Di}, and the coefficient of moment around the x-axis, c_{Mx}, of the wing.

We want to study the effect of the twist on this airfoil. For this reason, we consider a second airfoil that is exactly equal to the first one, but with a geometric twist:

$$\varepsilon(\theta) = 8\ \delta \left(\frac{y}{b}\right)^2$$

5. Calculate the twist as a function of θ.
6. Calculate the angle of attack of each wing section with respect to the zero-lift line of the section.
7. Calculate the non-dimensional circulation distribution on the new wing.
8. Calculate the lift coefficient, c_L, the induced drag coefficient, c_{Di}, and the coefficient of moment around the x-axis, c_{Mx}, of the wing. Compare them to the values of the first wing.

Problem 2:
Consider a rectangular wing with span b and large aspect ratio $\Lambda >> 1$, immersed on a uniform flow with freestream velocity U_∞, density ρ, and assume the induced downwash velocity along the span direction is:

$$\frac{w(y)}{U_\infty} = -\varepsilon \left[1 + 4\frac{y}{b} + 16\left(\frac{y}{b}\right)^2\right], \quad \varepsilon << 1$$

1. Obtain the value of the lift L of the wing.
2. Determine the induced drag D_i on the wing.
3. Calculate the roll moment about the x-axis M_x.