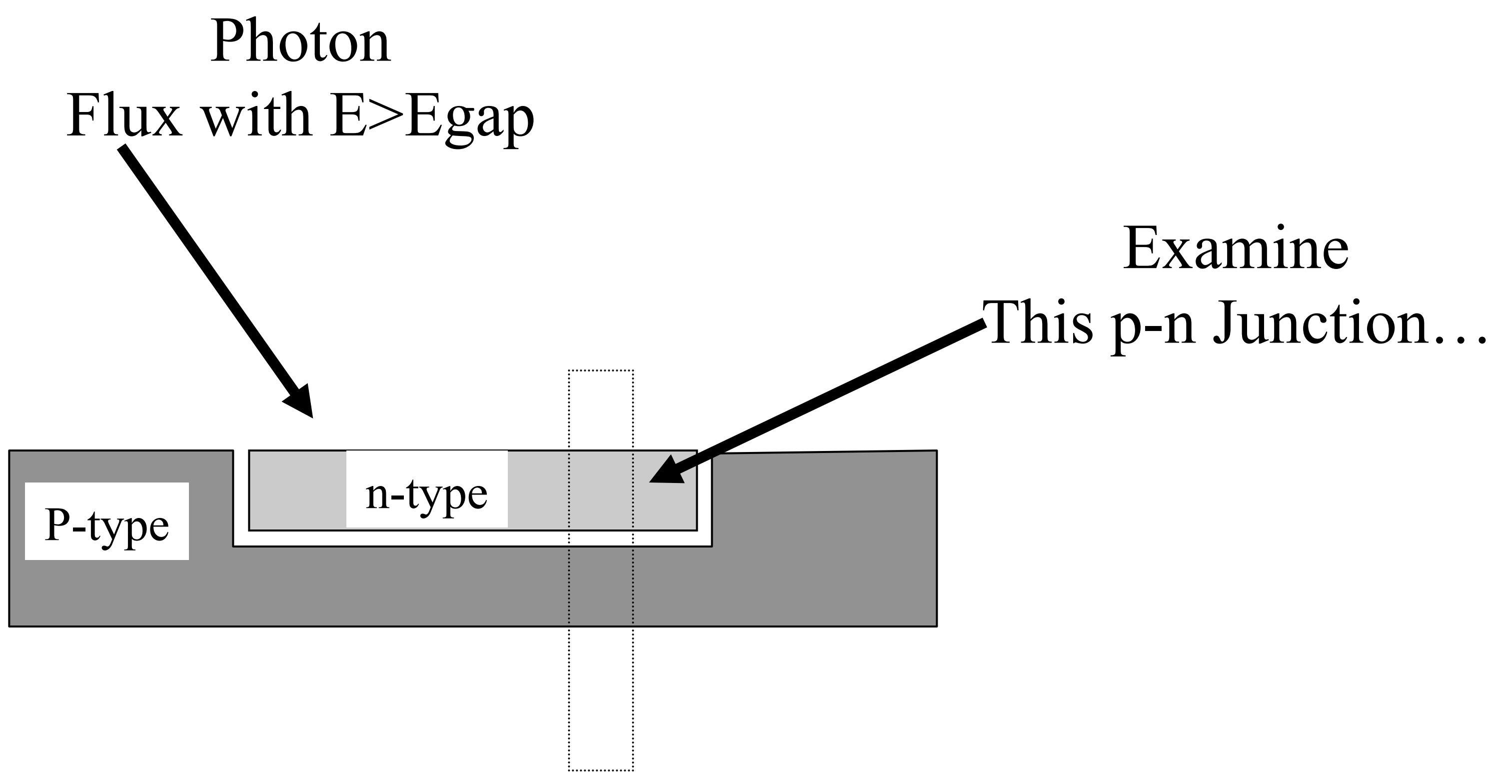
Quantitative Model of Unilluminated Diode – part I

G.R. Tynan

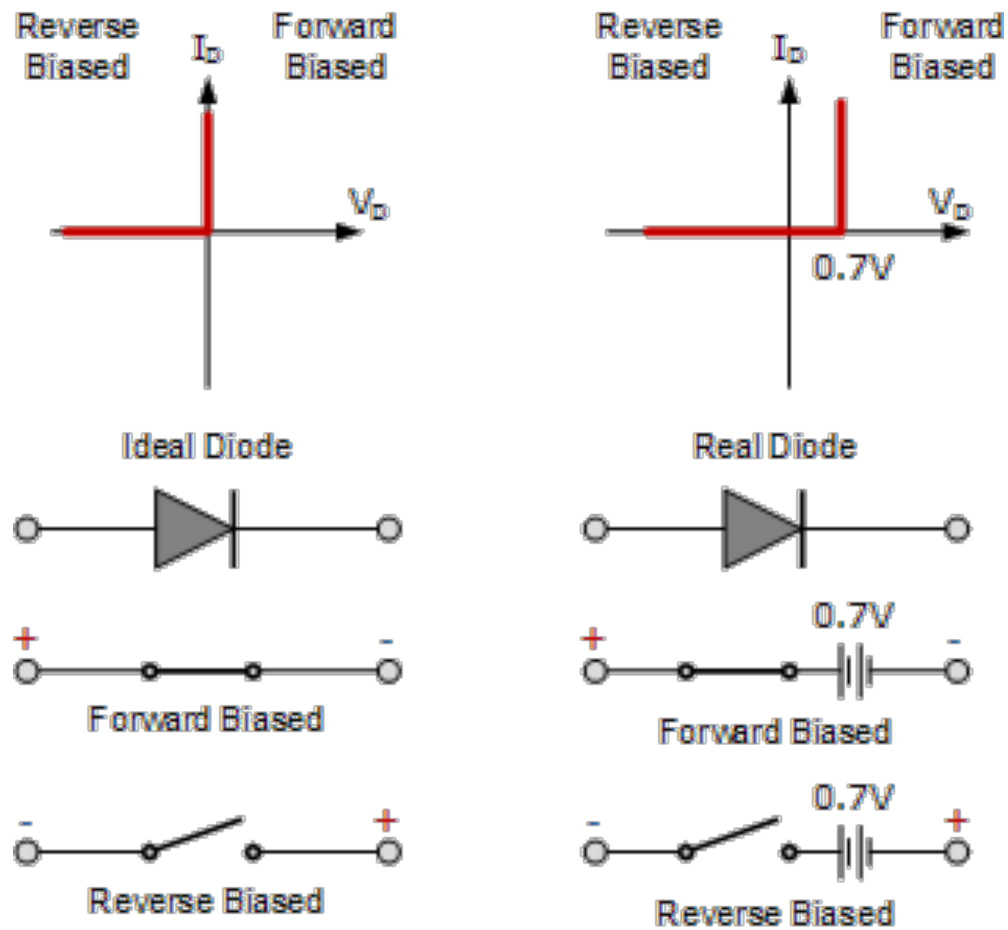
UC San Diego MAE 119

Lecture Notes

A Solar PV Cell is just a p-n junction ("diode") illuminated by light….

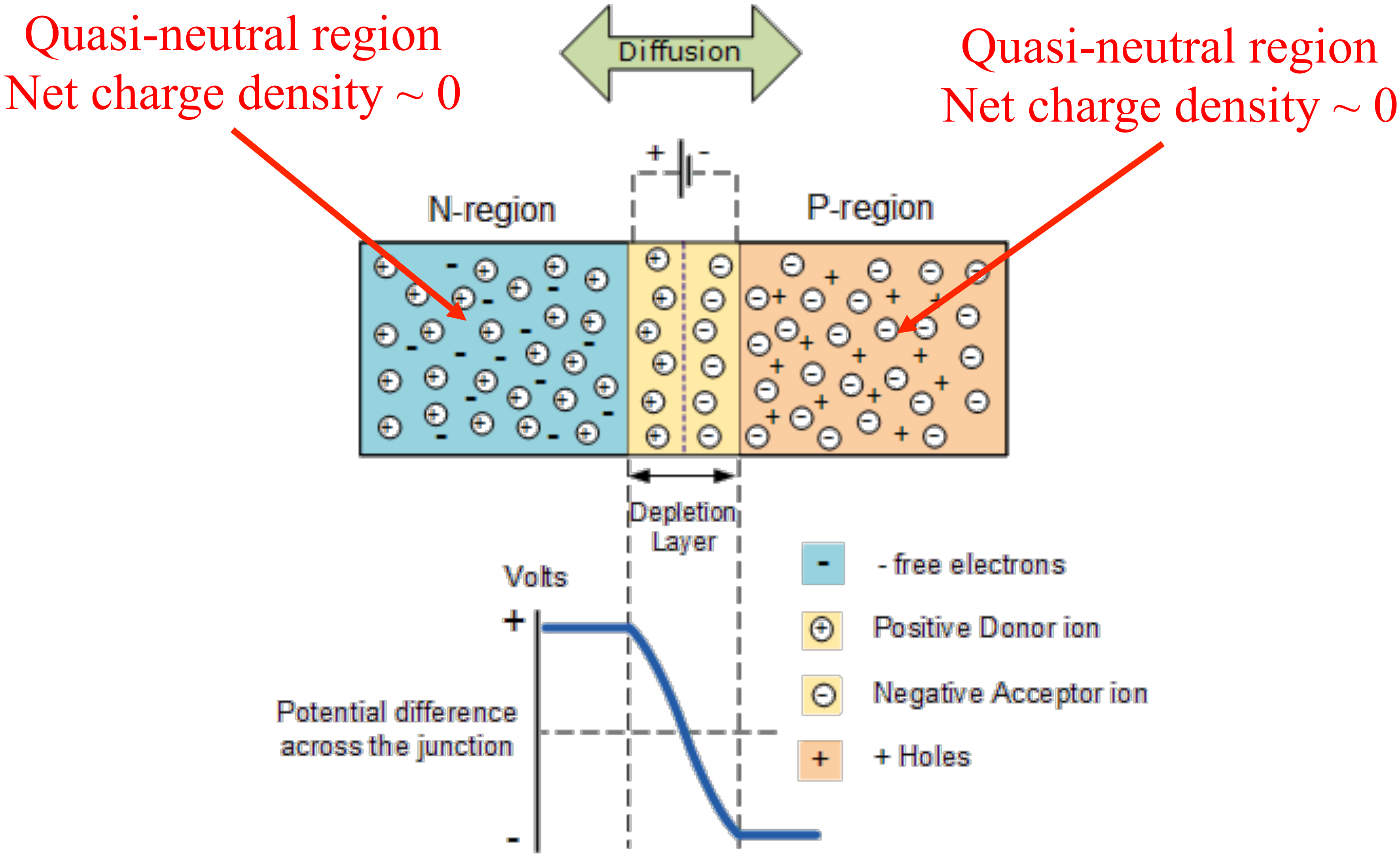


Diode current-voltage characteristics

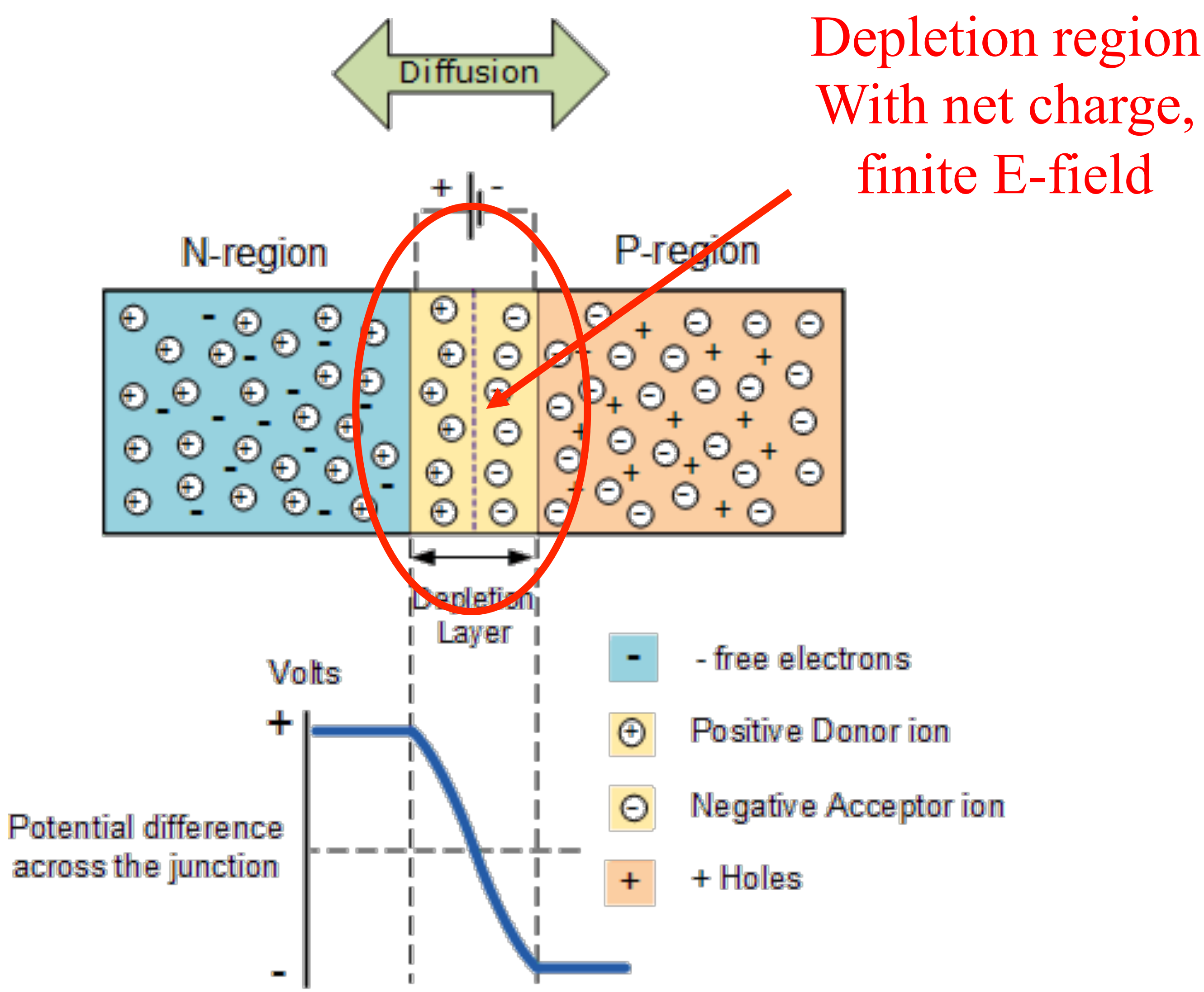


http://www.electronics-tutorials.ws/diode

Key elements: Quasineutral regions



Key elements: Depletion region



What is needed for quantitative model of this diode?

- What are the electron and hole densities?
- How do they move within the device?
  - Diffusion due to concentration gradients
  - Motion under action of electric fields
- Given these items, what is the current in the device as a function of voltage across device?

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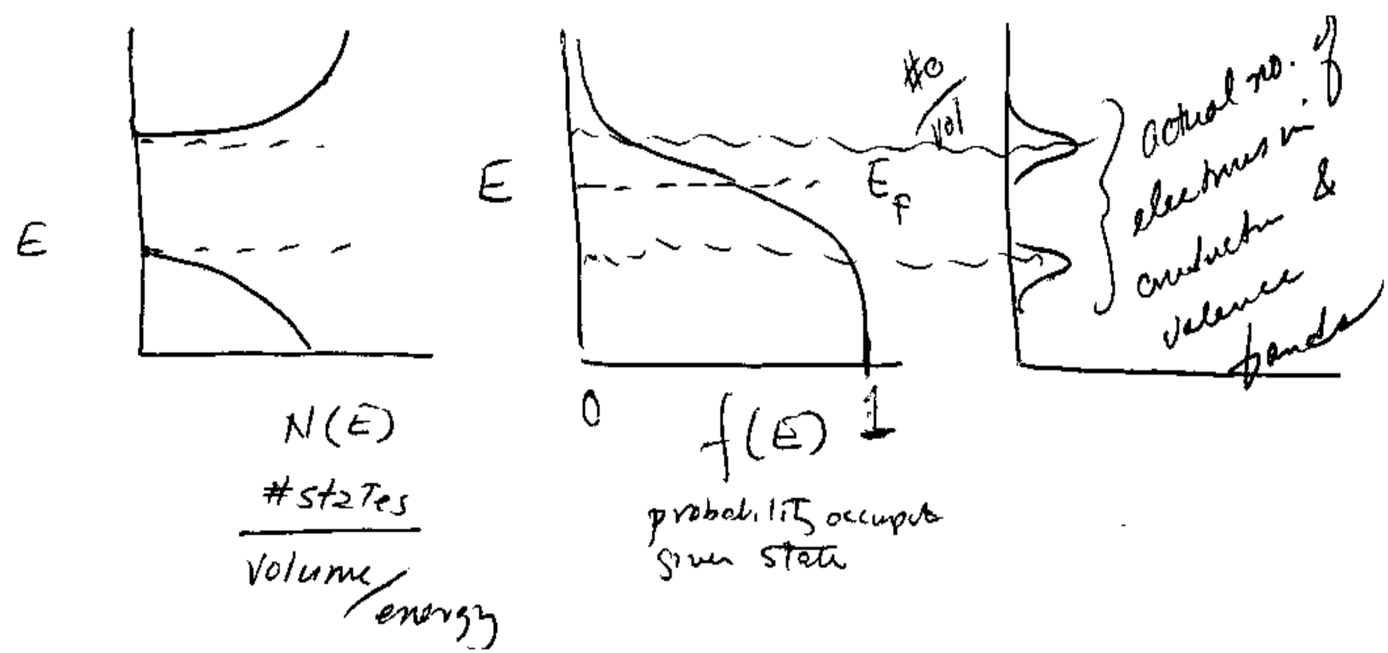
Determining Charge Carrier Densities

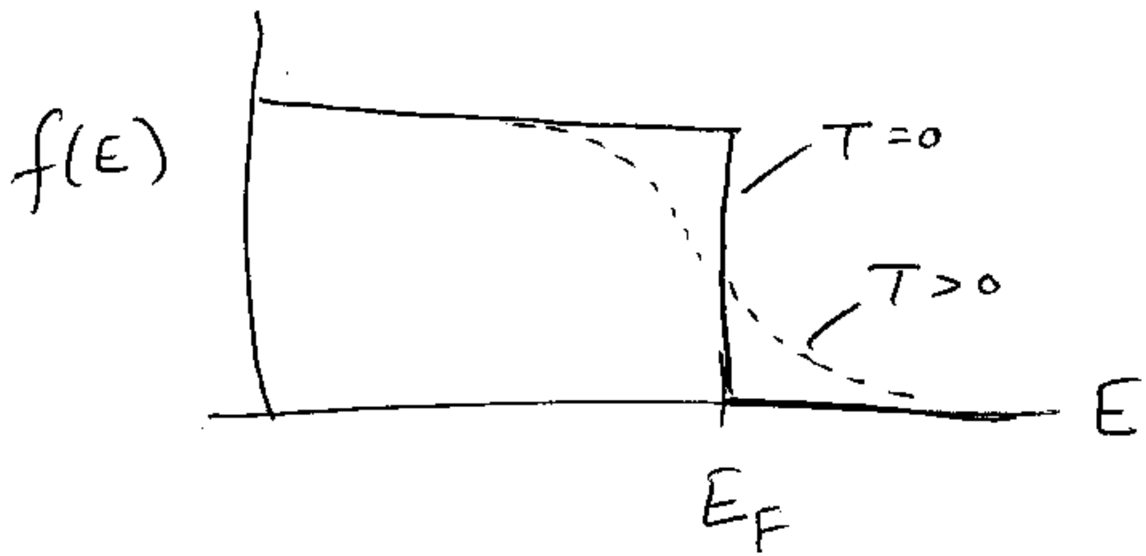
- Find number of mobile electrons/volume in conduction band from
  - Density of allowed states, N(E) and
  - Energy distribution of e and h, f(E):



Charge Carrier Energy Distribution

- Charge carrier particle energies follow Fermi-Dirac probability distribution

$$f(E) = \frac{1}{(exp^{\frac{E-E_F}{kT}}) + 1}$$



E_F = “Fermi Energy” characteristic upper limit for energy
T = Temperature of particles
f(E)dE = probability of finding particle between (E, E+dE)

Density of allowed charge carrier states

- Conduction Band Allowed State Density (states/vol-energy)

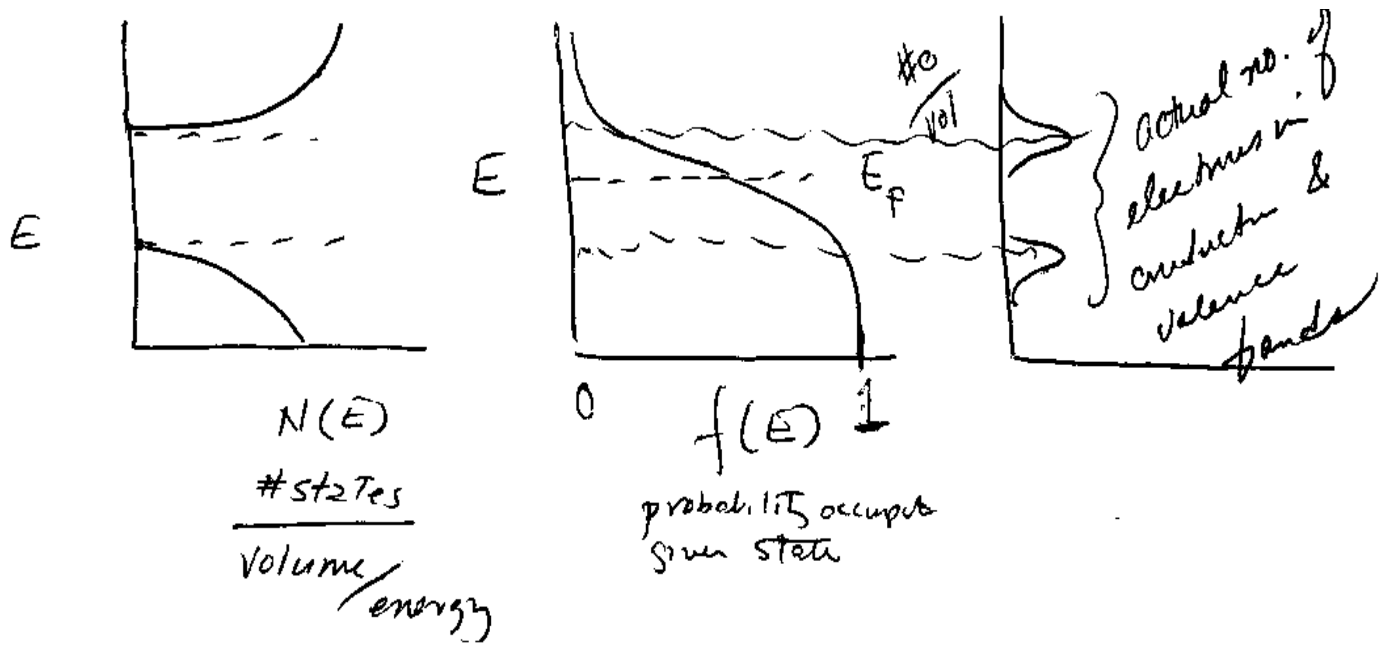
$$N_C(E) = \frac{8\sqrt{2}\pi\left(m_e^*\right)^{3/2}}{h^3}\left(E - E_C\right)^{1/2} \quad ; E \geq E_C \qquad m_e^*/m_e^o = 1.08$$

- Valence Band Allowed State Density (states/vol-energy:

$$N_C(E) = \frac{8\sqrt{2}\pi\left(m_e^*\right)^{3/2}}{h^3}\left(E - E_C\right)^{1/2} \quad ; E \geq E_C \qquad m_h^*/m_e^o = 0.81$$

Mobile Charge Carriers found from product of N(E)f(E):

- Find number of mobile electrons/volume in conduction band from N(E) and f(E):



Mobile Charge Carrier Density:

- Number of mobile electrons/volume in conduction band:

$$n = \int_{E_C}^{E_c^{\max}} f\left(E\right) N\left(E\right) dE \qquad E \geq E_C$$

- Result:

$$n = N_C \exp\left[\left(E_F - E_C\right)/kT\right]$$

$$N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

Mobile Charge Carrier Densities

- Number of mobile holes/volume in valence band:

$$p = \int_0^{E_V} N_V(E) f(E)\, dE$$

- Result:

$$p = N_V \exp\left[\left(E_V - E_F\right) / kT\right]$$

$$N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

A few useful definitions:

- Useful to define “intrinsic concentration, n_i :

$$n_i^2 = np = N_C N_V \exp\left[\left(E_V - E_C\right) / k_B T\right]$$

- Since n=p in a pure semiconductor can write $N_V \exp\left[\left(E_V - E_F\right)/kT\right] = N_C \exp\left[\left(E_F - E_C\right)/kT\right]$
- Which gives:

$$E_F = \frac{E_c + E_V}{2} + \frac{kT}{2}\ln\left(\frac{N_V}{N_C}\right)$$

Charge Carrier Densities – *n-type doped* semiconductors

- Charge neutrality tells us

$$p + n + N_D^+ - N_A^- = 0$$

- For n-type doped material

$$n = N_D^+ \quad and \quad N_D^+ \approx N_D$$

- And therefore in n-type material

$$p = n - N_D^+ << n$$

Charge Carrier Densities – *p-type doped* semiconductors

- Charge neutrality tells us

$$p + n + N_D^+ - N_A^- = 0$$

- For p-type doped material

$$p = N_A^- \quad and \quad N_A^- \approx N_A$$

- And therefore in p-type material

$$n = p - N_A^- << p$$

What is needed for quantitative model of this diode?

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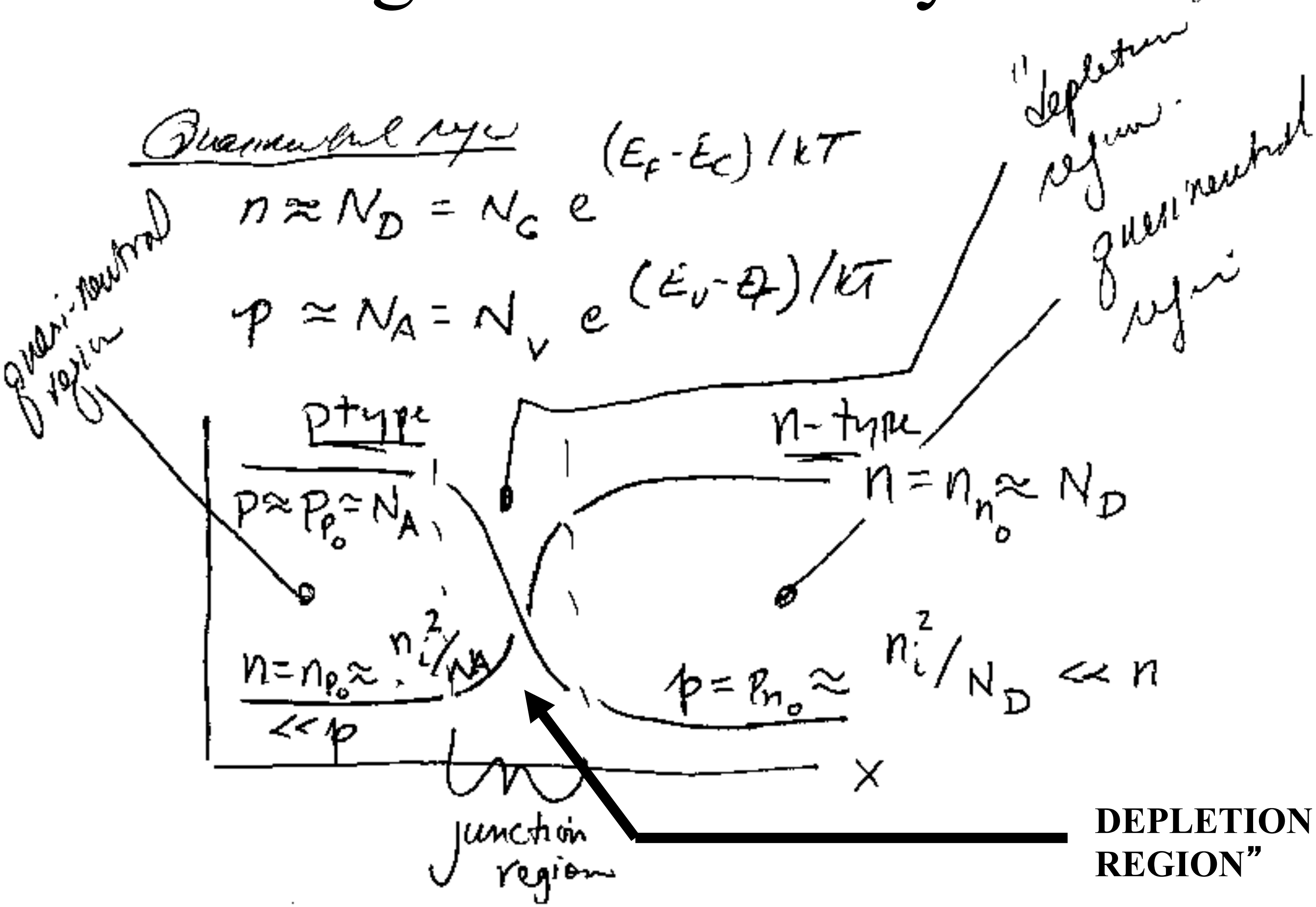
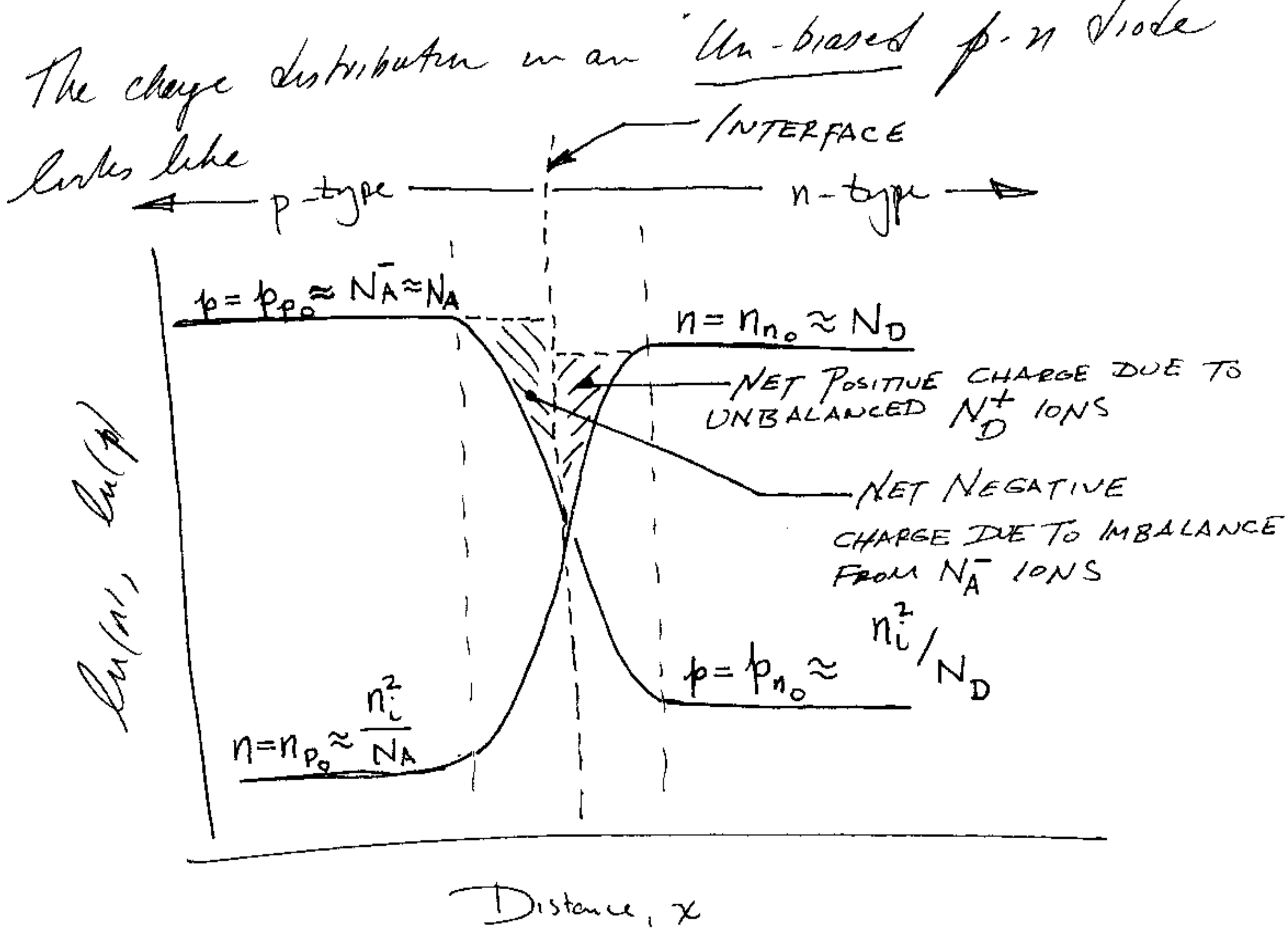
Resulting carrier density distribution



EXAMINE Depletion Region IN MORE DETAIL…

Charge Distribution Across Unbiased p-n diode



Idealized Model of Charge Distribution Across Depletion Region:

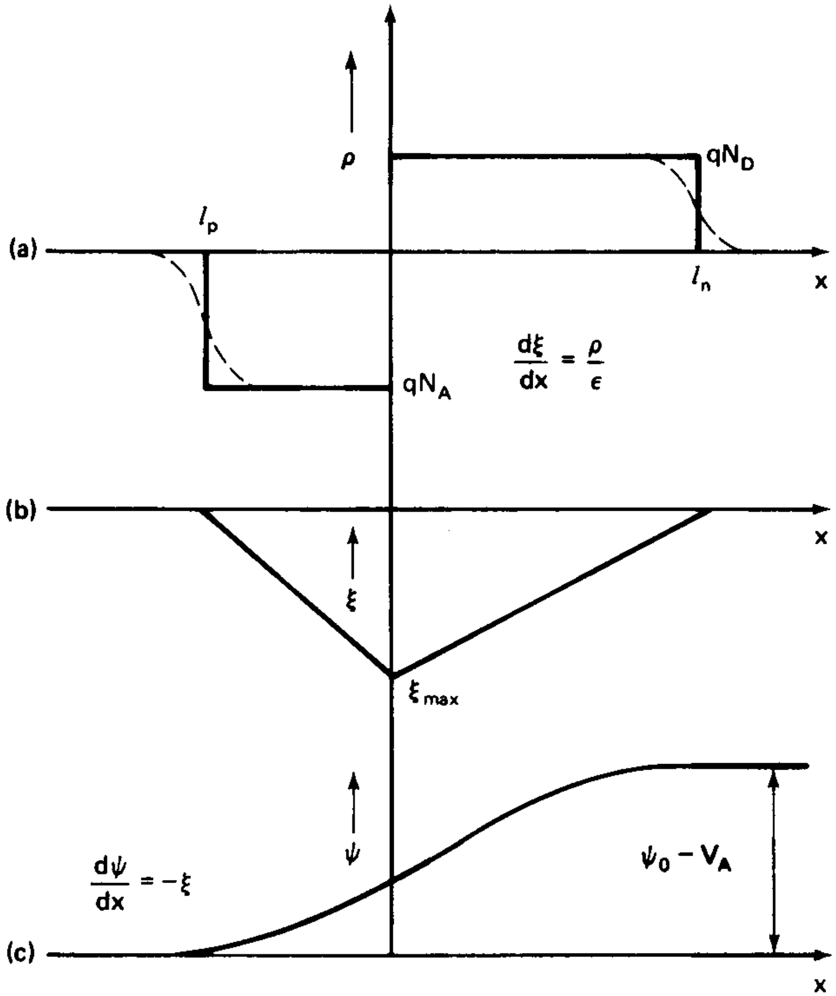


Figure 4.5. (a) Space-charge density corresponding to Fig. 4.4. The dashed line shows the actual distribution while the solid line shows the assumed distribution in the depletion approximation. (b) Corresponding electrical field strength. (c) Corresponding potential distribution.

Can Find Electric Potential in Depletion Region:

Charge Distribution From Semiconductor Material Theory

E-field from Poisson's Equation

Potential Profile by Integrating E-field

Poisson's eq'n relates charge density to E field:

Poisson's Equation: $$\frac{dE}{dx} = \frac{q}{\varepsilon}(p - n + N_D^+ - N_A^-)$$

Donor/Acceptor Ion Density $N_D^+ \approx N_D$ $N_A^- \approx N_A$

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p, n distribution given by step functions seen in previous viewgraph

Integrate Poisson's Equation to Find E(x);
Integrate Again to find potential distribution

Idealized Model of Depletion Region:

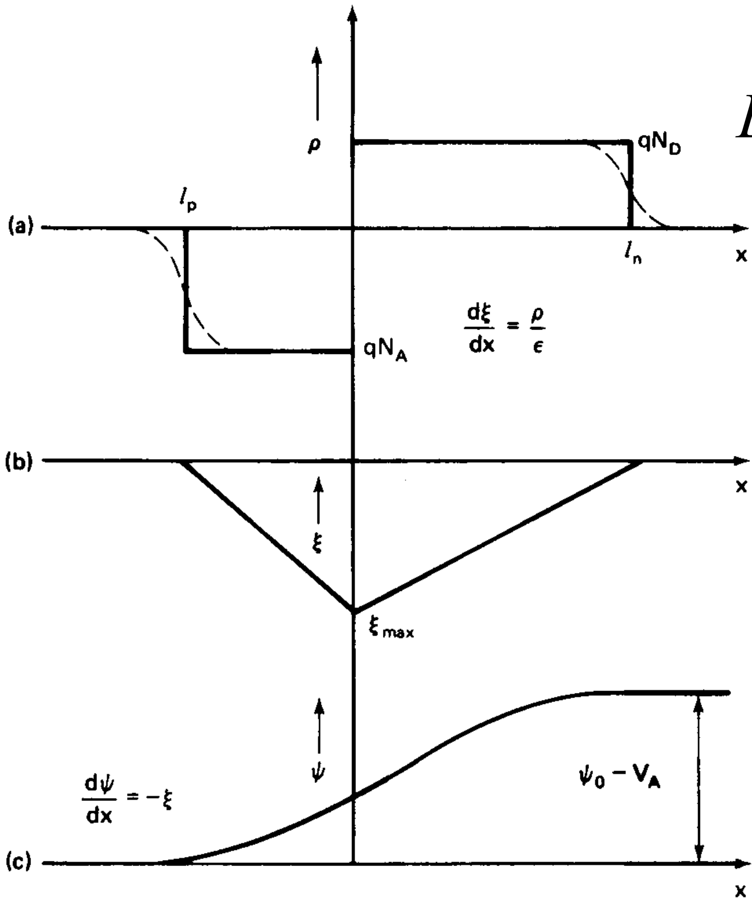

Figure 4.5. (a) Space-charge density corresponding to Fig. 4.4. The dashed line shows the actual distribution while the solid line shows the assumed distribution in the depletion approximation. (b) Corresponding electrical field strength. (c) Corresponding potential distribution.

RESULT:

$$E_{\max} = -\left[\frac{2q}{\varepsilon}\left(\psi_0 - V_{ext}\right)\bigg/\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\right]^{1/2}$$

$$w = l_p + l_n = -\left[\frac{2\varepsilon}{q}\left(\psi_0 - V_{ext}\right)\bigg/\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\right]^{1/2}$$

$$l_p = W\frac{N_D}{N_A + N_D}$$

$$l_n = W\frac{N_A}{N_A + N_D}$$

Carrier concentration at <u>edges of un-biased junction</u>

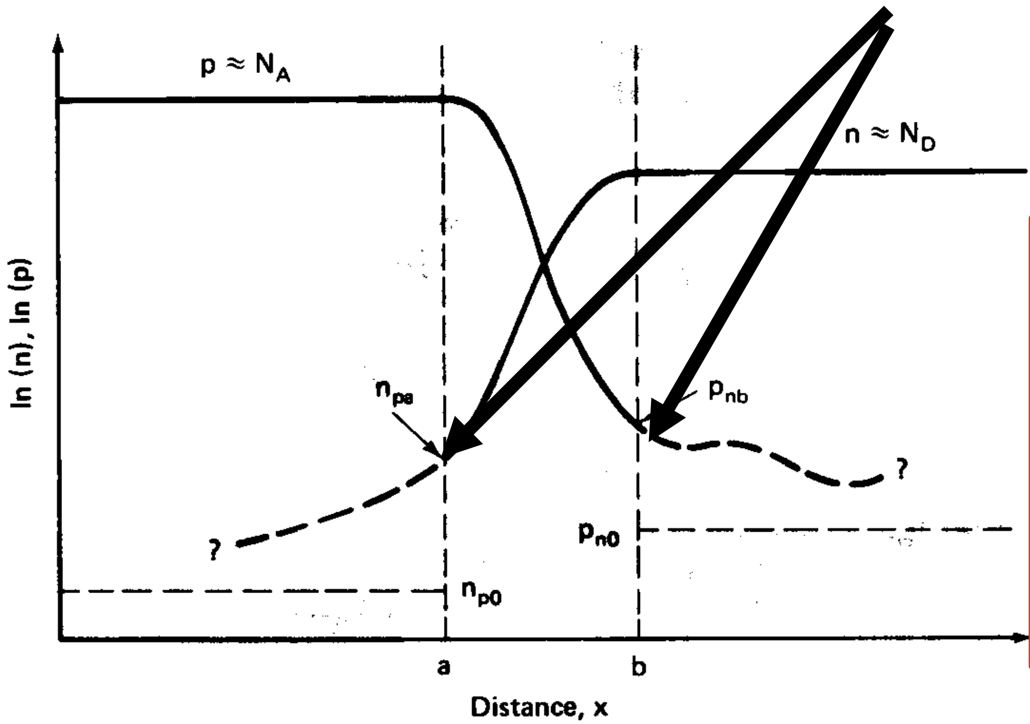

Figure 4.7. Plot of carrier concentrations when a voltage is applied to the *p-n* junction. In the text, expressions are found for the minority carrier concentrations n_{pa} and p_{nb} at the edge of the junction depletion region. Subsequently, the precise form of the distributions shown dashed are also calculated.

With NO Ext. Voltage, V_{ext}=0:

$$p_{nb} = p_{n_0} = p_{p_0} e(\frac{-q\psi o}{kT}) \approx \frac{n_i^2}{N_D}$$

$$n_{b_a} = n_{p_0} = n_{n_0} e\left(\frac{-q\psi o}{kT}\right) \approx \frac{n_i^2}{N_A}$$

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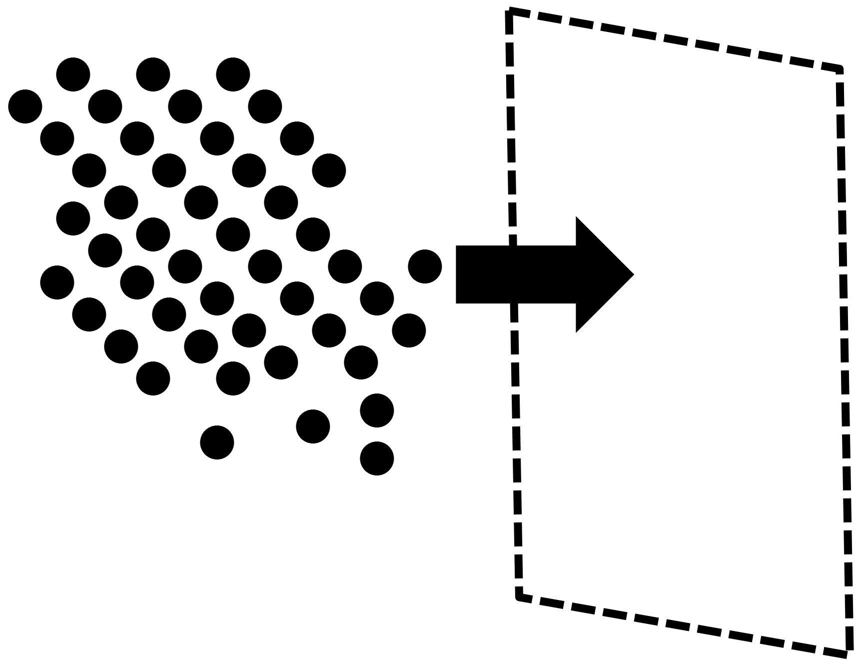
Question: What happens when we apply an external voltage to this diode?

In order to answer, need to first define current density, J (Amps/m^2)…

Need to define current density, J:

Consider a collection of *n (p)* electrons/unit volume (holes/unit volume) that are moving towards the right



Q: How many charges pass thru the surface per unit area/unit time?
A: This is defined as the electron (hole) **current density**, J_e (J_h)

Can be caused by **E-field** and/or by **diffusion**

Basic Equations of Semiconductor Device Physics

Poisson's Equation: $$\frac{dE}{dx} = q(p - n + N_D^+ - N_A^-)$$

Donor/Acceptor Ion Density $N_D^+ \approx N_D$ $N_A^- \approx N_A$

Current Transport

$$J_e = q\mu_e nE + qD_e \frac{dn}{dx}$$

$$J_h = q\mu_{he} pE - qD_e \frac{dp}{dx}$$

S.S. Charge Conservation:

$$\frac{1}{q}\frac{dJ_e}{dx} = U - G$$

$$\frac{1}{q}\frac{dJ_h}{dx} = -(U - G)$$

U~Source
G~Sink

Question: What happens when we apply an external voltage to this diode?

A: Currents can begin to flow…

How does charge distribution respond to V_{bias}?

In general current density depends on E and gradient, i.e.

$$J_h = qp\mu_h E - qD_h \, {\partial p}/{\partial x}$$

We will simplify analysis by assuming that

$$qp\mu_h E \approx qD_h \frac{dp}{dx} \quad \textbf{\textit{even if}} \quad J_h \neq 0$$

Now…

$$D_e = \frac{kT}{q}\mu_e$$

$$D_h = \frac{kT}{q}\mu_h$$

$$\Rightarrow \quad E \approx \frac{kT}{q}\frac{1}{p}\frac{dp}{dx}$$

And since $E = -{\partial \phi}/{\partial x}$ we can integrate to find potential drop across depletion region…

How does charge distribution respond to V_{bias}?

And since $E = -\partial\phi / \partial x$ we can integrate to find potential drop across depletion region…

$$\phi_\circ - V_a = +\frac{kT}{q}\ln\frac{p_{p_a}}{p_{n_b}}$$

Which can be re-arranged to give:

$$p_{n_b} = p_{p_a}\exp\left[\frac{q\phi_\circ}{kT}\right]\exp\left[\frac{qV_a}{kT}\right]$$

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Get similar result for electrons…

How does charge distribution respond to V_{bias}?

And since $E = -\partial\phi / \partial x$ we can integrate to find potential drop across depletion region…

$$\phi_\circ - V_a = +\frac{kT}{q}\ln\frac{p_{p_a}}{p_{n_b}}$$

Exponential Increase w/ V_a>0 !

Which can be re-arranged to give:

$$p_{n_b} = p_{p_a}\exp\left[\frac{q\phi_\circ}{kT}\right]\exp\left[\frac{qV_a}{kT}\right]$$

p_{n_0} Minority carrier density w/o ext. bias

Get similar result for electrons… THIS IS A KEY RESULT!

Forward Bias Increases Minority Charge Carrier Density at Edge of Quasineutral region:

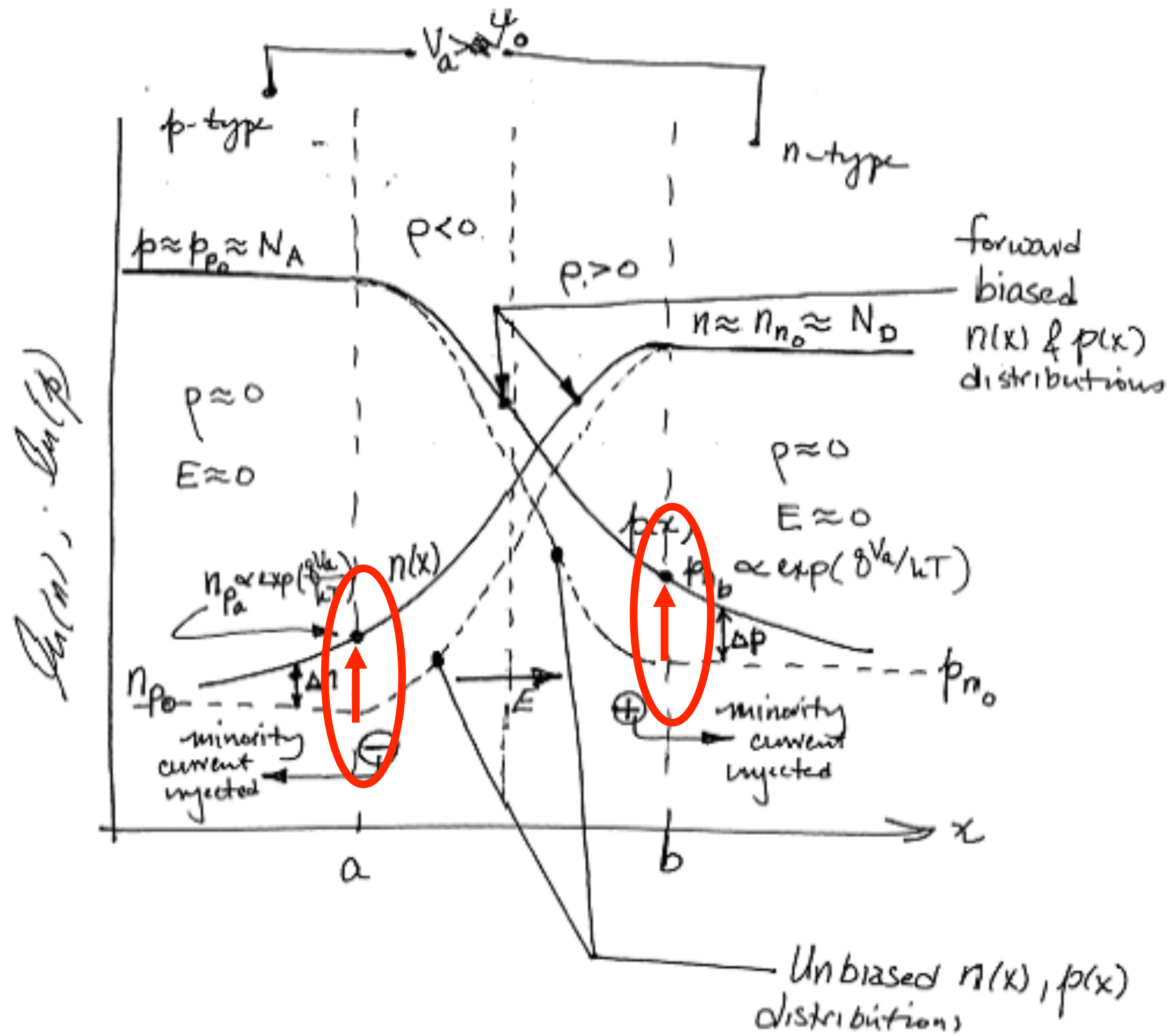
Concentration
Of MINORITY
CARRIERS at
Edge of Depletion Region
INCREASES
EXPONENTIALLY
W/ Ext. Bias

$$p_{n_b} = p_{n_0} e^{qV_{app}/kT} = \frac{n_i^2}{n_D} e^{qV_{app}/kT}$$

$$n_{p_a} = n_{p_0} e^{qV_{app}/kT} = \frac{n_i^2}{N_A} e^{qV_{app}/kT}$$

➔KNOWN AS MINORITY
➔CARRIER INJECTION

Minority Carrier Density at edge of quasineutral region increases **EXPONENTIALLY** forward bias



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