Maximum Theoretical Efficiency of PV Cells

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Lecture Notes
Band Theory of Solids: From Single Atoms...to Solid Crystals

- Isolated Li atom (conducting metal)
  - Has well-defined, isolated allowable electron energy levels

- N isolated atoms
  - N x isolated atom levels

- Strongly interacting Li atoms
  - Interaction shifts (or splits) individual energy bands into isolated regions separated by forbidden bands
Next, consider $N$ interacting sodium atoms at 0 deg K

- Electrons in config $1s^2 2s^2 2p^6 3s^1$
- Shells filled to 3s, which has 1 electron
Next, consider N interacting sodium atoms w/ T>0

- Electrons in config 1s²2s²2p⁶3s¹
- Shells filled to 3s, which has 1 electron
- This Valence electron is weakly bound =>>> if T High enough can move to mobile state ⇒ conductor!
Band Theory of Solids: Insulators

- Carbon in Diamond Form
  - Electrons in $1s^22s^22p^2$ State
  - $2p$ band has $2N$ electrons, but $6N$ states
  - BUT… crystal structure splits $2p$ into two distinct bands
  - **BAND GAP is $\sim 6 \text{ eV}$**
  - $>>$ Temperature ($\sim 0.02-0.1 \text{ eV}$)
  - **Thus…Diamond is An Insulator**

<table>
<thead>
<tr>
<th>Number of available states</th>
<th>Number of electrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4N$</td>
<td>0</td>
</tr>
<tr>
<td>$2N$</td>
<td>$2N$</td>
</tr>
<tr>
<td>$2N$</td>
<td>$2N$</td>
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<td>$2N$</td>
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</tbody>
</table>

Lower levels: VALENCE BAND
Upper levels: CONDUCTION BAND
Band Theory of Solids: Semiconductors

- Some crystalline materials have smaller band-gap energy
- At low temperatures behave like insulators
  - $E_{bg} \sim 1\text{eV} >> \text{Temperature}$
- With an electric field
  - Electrons gain energy
  - Can move into upper (conduction) band
Si as a Semiconductor Material

A Silicon Atom, Atomic number = "14"

Silicon atom showing 4 electrons in its outer valence shell (m)

Co-valent Bonds

Valence Shell (m)

Si

Si

Si

Si

Shared Electrons

Silicon Crystal Lattice

http://www.electronics-tutorials.ws/diode
N-type Semiconductor Materials

N-type Si has an extra electron for each dopant atom,
This electron is mobile

http://www.electronics-tutorials.ws/diode
P-type Si has a “hole” (i.e., a missing electron) that acts like a mobile positive charge.

http://www.electronics-tutorials.ws/diode
Dopants create allowed energy states between the pure material valence and conduction bands

- Pure semiconductor matl’s conduction and valence bands separated by $E_{\text{gap}}$
- In pure materials this gap has no allowed states -> no particles in these energy ranges
- IF ADD donor or acceptor impurities then this creates allowed states between the pure-material conduction & valence bands
Physics of maximum theoretical efficiency

• Key Concepts
  – Photon Energy Spectrum
  – Charge Carrier Generation Via Photon Absorption
  – Photon flux & relation to energy spectrum
  – Estimating maximum possible efficiency
  – What does a real cell look like?
Physics of maximum theoretical efficiency

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Recall blackbody spectrum:

Blackbody spectrum:

\[ I(\nu) = \frac{h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1} \]

Total intensity:

\[ I_{\text{tot}} = \int_0^\infty I(\nu)d\nu \]
Physics of maximum theoretical efficiency

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Light absorption can (sometimes) create mobile e-h pairs:

(i) forbidden

\( h\nu < E_{\text{gap}} \)

\( E = h\nu < E_{\text{gap}} \)

\( E_{\text{gap}} \)

\( E_{c} \)

\( E_{v} \)

\( \text{photon w/ } h\nu > E_{\text{gap}} \)

\( \text{photon cannot dislodge } e^- \text{ from V-band} \)
What portion of spectrum has photons w/ enough energy?

\[ \gamma = \frac{E_{\text{gap}}}{h} \]

These photons are lost as heat.

These photons have
\[ h\gamma > E_{\text{gap}} \]
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Relation of photon flux to Intensity:

Collection of \( n \) photons/unit volume
With frequency in range \((v, v+dv)\)

Q: How many photons pass thru the surface per unit area/unit time?
Relation of photon flux to Intensity:

Collection of $n$ photons/unit volume
With frequency in range $(v, v+dv)$

Q: How many photons pass thru the surface per unit area/unit time?
A: This is the photon flux, $\phi(v) = n(v)c$
Relation of photon flux to Intensity:

Collection of n photons/unit volume
With frequency in range \((v, v+dv)\)

Q: If each photon has energy \(E=\hbar v\), how much energy passes thru surface per unit area and per unit time?
Relation of photon flux to Intensity:

Collection of $n$ photons/unit volume

With frequency in range $(\nu, \nu+\nu)$

Q: If each photon has energy $E=\hbar \nu$, how much energy passes thru surface per unit area and per unit time?

A: Energy per unit area/unit time is INTENSITY,

$$I(\nu) = h\nu \phi(\nu)$$
Photon flux of blackbody spectrum:

Blackbody spectrum:  

Total flux (photons/unit area-time):

\[
\phi(\nu) = \frac{\nu^2}{c^2 \exp(\frac{h\nu}{kT}) - 1}
\]

With \((\nu, \nu+\text{d}\nu)\)
Photon flux of blackbody spectrum:

$$\phi_{gap} = \int_{v_{gap}}^{\infty} \phi(v) \, dv$$

Photon flux with $E > E_{gap}$

Each of these Photons produce Charge carrier pair With potential energy $E_{gap}$

$$\phi(v) \quad \text{(photons / m}^2 \text{- Hz)}$$

$$v_{gap} = \frac{E_{gap}}{h}$$

frequency $v \quad \text{(Hz)}$
Can now estimate max efficiency:

Power produced by e-h pair creation:

\[ P_{\text{max}} = \phi_{\text{gap}} E_{\text{gap}} \]

Where \( \phi_{\text{gap}} = \int_{v_{\text{gap}}}^{\infty} \phi(v) dv \)

Maximum incident power (per unit area):

\[ I_{\text{tot}} = \int_{0}^{\infty} I(v) dv \]

Maximum possible efficiency is the ratio of these two:

\[ \eta_{\text{max}} = \frac{P_{\text{max}}}{I_{\text{tot}}} \]
Max. PV Cell Efficiency:

Can recast as an integral:

\[ \eta_{\text{max}} = \frac{15}{\pi^4} \xi_0 \int_{\xi_0}^{\infty} \frac{x^2}{e^x - 1} \, dx \]

where \( \xi_0 = \frac{qE_{\text{gap}}}{k_B T_{bb}} \)

For Si with \( E_{\text{gap}} \sim 1.1 \text{ eV} \) and \( T_{bb} \sim 6000 \text{ K} \)

\[ \eta_{\text{max}} \sim 0.44 \]
A Solar PV Cell is just a p-n junction ("diode") illuminated by light....

Photon Flux with $E > E_{\text{gap}}$

Examine This p-n Junction...