Solutions to Homework 1

January 14, 2017

1 Problem 1 – Energy Consumption

World energy consumption in 2010 was about 500 quadrillion BTUs. That is $E = 500 \times 10^{15}$ BTUs which also equal $E = 500 \times 10^{15} \times 1055 = 5.275 \times 10^{20}$ J. (Some of following energy equivalents are from http://www.convert-me.com/en/convert/energy/)

a) One barrel of oil equivalent (BOE) is defined as equal to 5.8×10^6 BTUs. Thus $\frac{E}{BOE} = \frac{500 \times 10^{15}}{5.8 \times 10^6} = 8.62 \times 10^{10}$ Barrels.

b) One tonne of coal equivalent (TCE) is 2.778×10^7 BTUs. $\frac{E}{TCE} = \frac{500 \times 10^{15}}{2.778 \times 10^7} = 1.80 \times 10^{10}$ Tons.

c) One standard cubic foot of natural gas can produce 1028 BTUs. $\frac{500 \times 10^{15}}{1028} = 4.86 \times 10^{14}$ standard cubic feet.

d) To elevate one liter of water from sea level to 1000 meters above needs

$$
\begin{aligned}
E_1 &= \rho V g h \\
&= 1 \times 10^3 (\frac{kg}{m^3}) \times 10^{-3} (m^3) \times 9.8 (\frac{m^2}{s}) \times 1000 (m) \\
&= 9.8 \times 10^3 \text{ J.}
\end{aligned}
$$

$\frac{E}{E_1} = \frac{5.275 \times 10^{20}}{9.8 \times 10^3} = 5.38 \times 10^{16}$ Liters.

e) Specific enthalpy of saturated steam is about $H_v^{200} = 2792\,\text{kJ/kg}$ at 200 °C. Specific enthalpy of water is about $H_l^{25} = 105\,\text{kJ/kg}$ at room temperature (25 °C). $\dfrac{E}{H_v^{200} - H_l^{25}} = 1.96 \times 10^{14}\,\text{kg}$. (Enthalpy values are from http://www.tlv.com/global/TI/calculator/steam-table-temperature.html)

f) Specific enthalpy of hot water is about $H_l^{200} = 852\,\text{kJ/kg}$ at 200 °C. Specific enthalpy of water is about $H_l^{25} = 105\,\text{kJ/kg}$ at room temperature (25 °C). $\dfrac{E}{H_l^{200} - H_l^{25}} = 7.06 \times 10^{14}\,\text{kg}$. (Enthalpy values are from http://www.tlv.com/global/TI/calculator/steam-table-temperature.html)

g) Heat released from granite of unit volume by cooling it from 250 °C to room temperature (25 °C) is

$$
\begin{aligned}
q/V &= \rho c_p \Delta T \\
&= 2400\,\text{kg/m}^3 \times 790\,\text{J/(kg °C)} \times (250 - 25)\,\text{°C}.
\end{aligned}
$$

Thus $V = \dfrac{E}{q/V} = \dfrac{5.275 \times 10^{20}}{2400 \times 790 \times (250 - 25)} = 1.24 \times 10^{12}\,\text{m}^3$. (http://www.engineeringtoolbox.com/sensible-heat-storage-d_1217.html)

h) Typical fission events release about 200 MeV of energy for each fission event, for example ${}^{235}_{92}U + n \longrightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2n + 200\,\text{MeV}$. Therefore, energy released from one kilogram uranium-235 is $E_1 = \dfrac{200(MeV) \times 1.6 \times 10^{-19}(J/MeV)}{235 \times 1.66 \times 10^{-27}(kg)} = 8.20 \times 10^{13}\,\text{J/kg}$. Mass of fissile uranium is then $m = \dfrac{E}{E_1} = 6.44 \times 10^{6}\,\text{kg}$. (https://en.wikipedia.org/wiki/Nuclear_fission#Output)

2 Problem 2 – CO_2 Emissions

a) The CO_2 emissions per barrel crude oil is about 0.43 tons per barrel. (https://www.epa.gov/energy/ghg-equivalencies-calculator-calculations-and-references.) $m_{CO_2} = 0.43 \times 8.62 \times 10^{10} = 3.7 \times 10^{10}$ tons.

b) One pound of coal produces 9.37×10^{-4} tons of CO_2. $m_{CO_2} = 9.37 \times 10^{-4} \times 1.80 \times 10^{10} \times 2204.6 = 3.72 \times 10^{10}$ tons.

c) One thousand standard cubic foot of natural gas produces 0.054717 tons of CO_2. $m_{CO_2} = 0.054717 \times 4.86 \times 10^{14}/1000 = 2.67 \times 10^{10}$ tons.

d) 0

e) 0

f) 0

g) 0

h) 0

3 Problem 3 – Individual Energy Consumption

Assume one consumes 1000 kW h electricity and drives 1×10^4 miles per year. The energy consumed per year is $E = 1000 \times 3.6 \times 10^6\,\mathrm{J} + 1 \times 10^4\,\mathrm{mile}/(25\,\mathrm{mpg}) \times 121\,\mathrm{MJ/gallon} = 5.2 \times 10^{10}\,\mathrm{J}$.

- One barrel of oil produces 6.12×10^9 J. One consumes $5.2 \times 10^{10}/6.12 \times 10^9 = 8.4$ barrels of oil. This leads to CO_2 emissions of $0.43 \times 8.4 = 3.61$ tons.
- One ton of coal produces 2.93×10^{10} J. One consumes $5.2 \times 10^{10}/2.93 \times 10^{10} = 1.78$ tons of coal. This leads to CO_2 emissions of $9.37 \times 10^{-4} \times 1.78 \times 2204.6 = 3.68$ tons.
- One standard cubic foot of natural gas produces 1.09×10^6 J. One consumes $5.2 \times 10^{10}/1.09 \times 10^6 = 4.78 \times 10^4$ standard cubic feet of natural gas. This leads to CO_2 emissions of $0.054\,717 \times 4.78 \times 10^4/1000 = 2.62$ tons.

One's body typically consumes 2500 kcal every day. That is $2500\,\mathrm{kcal} \times 365 \times 4148\,\mathrm{J/kcal} = 3.79 \times 10^9$ J per year, which is comparable to the electricity energy one consumed every year.

4 Problem 4 – CO_2 Emission Expectation

4.1 Constant Energy Intensity

Assume the energy intensity keeps constant in the model and can be weighted as 2×10^7 J/dollar. The total energy demand, E_{tot} is then calculated by the multiplication of energy intensity and global economic activity (GDP). The CO_2 emissions can therefore be determined via $\frac{E_{tot}}{C_{\text{intensity}}}$, where $C_{\text{intensity}} = 50\,\text{MJ/kg}$. Results are plotted in Fig. 1. The code is shown below.

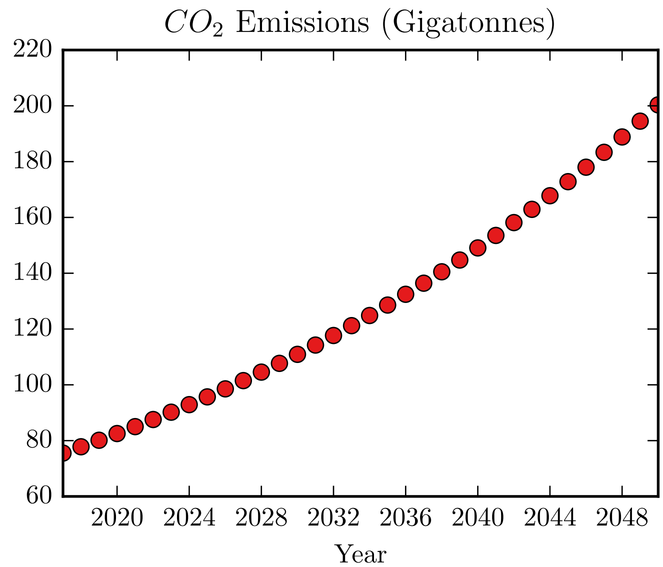

Figure 1: CO_2 emissions with constant energy intensity.

```
# Python code for Problem 4 (a).
import numpy as np
import matplotlib.pyplot as plt

year = np.asarray([x for x in range(2017, 2051)])
gdp_now = 5e13  # dollar
gdp_rate = 0.03
annual_energy_now = 15e12 * 365 * 24 * 3600  # joule
carbon_intensity = 50e6  # joule per kg of methane

gdp = np.asarray([gdp_now * (1 + gdp_rate) ** x
                  for x in range(1, len(year) + 1)])

energy_intensity = 2e7  # joule per dollar
annual_energy_demand = energy_intensity * gdp  # joule
co2_emission = annual_energy_demand / carbon_intensity * (44 / 12)
```

```
years = [str(x) for x in year]
fig = plt.figure(figsize=[4, 3])
plt.plot_date(years, co2_emission / 1e9 / 1e3)
plt.xlabel('Year')
```

4.2 Decreasing Energy Intensity

Assume the energy intensity decreases at 1% per year in the model with an initial value of 2×10^7 J/dollar. Repeating the calculations in the code below, we can get the results shown in Fig. 2.

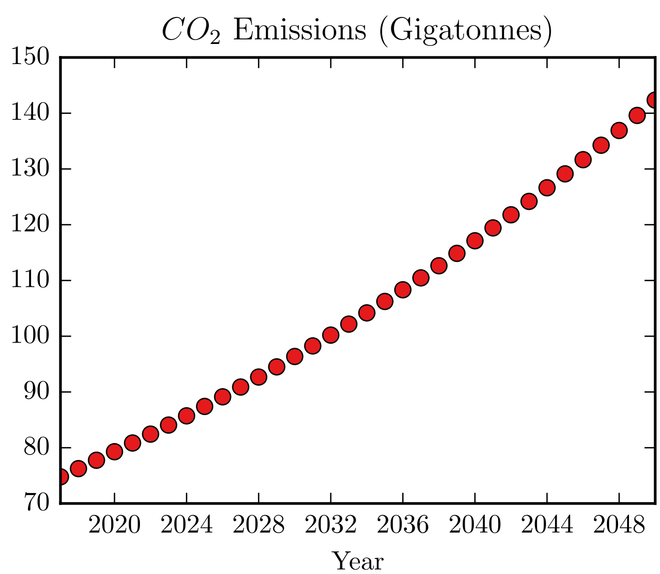

Figure 2: CO_2 emissions with decreasing energy intensity.

```
# Python code for Problem 4 (b).
import numpy as np
import matplotlib.pyplot as plt

year = np.asarray([x for x in range(2017, 2051)])
gdp_now = 5e13  # dollar
gdp_rate = 0.03
annual_energy_now = 15e12 * 365 * 24 * 3600  # joule
carbon_intensity = 50e6  # joule per kg of methane

gdp = np.asarray([gdp_now * (1 + gdp_rate) ** x
```

```python
                     for x in range(1, len(year) + 1)])

energy_intensity0 = 2e7  # joule per dollar
rate = 0.01
energy_intensity = np.asarray(
    [energy_intensity0 * (1 - rate) ** x for x in range(1, len(year) + 1)])
annual_energy_demand = energy_intensity * gdp  # joule
co2_emission = annual_energy_demand / carbon_intensity * (44 / 12)

years = [str(x) for x in year]
fig = plt.figure(figsize=[4, 3])
plt.plot_date(years, co2_emission / 1e9 / 1e3)
plt.xlabel('Year')
plt.title(u'$CO_2$ Emissions (Gigatonnes)')
```

4.3 Discuss the Results from these Two Models

Although being over simplified, both models yield roughly the same CO_2 emission trajectory as the much more complicated IPCC models. Decreasing the energy intensity of our economy (increasing efficiency) slows our emissions of greenhouse gas, but only delays the business as usual outcome. In other words, we cannot rely on increasing efficiency to solve our climate and energy problems. Clean energy techniques that do not rely on fossils fuels should be developed to resolve those problems.

5 Problem 5 – Human Index and Electricity Demand

As shown in Figure 1 in Pasternak's report, a HDI of 0.9 corresponds to an electricity consumption of at least 4000 kWh per capita per year. For 7×10^9 people in the world, the total electricity demand is at least $7 \times 10^9 \times 4000 = 2.8 \times 10^{13}$ kWh = 28 trillion kWh. The world electricity consumption is about 20 trillion kWh in 2014 (http://www.iea.org/publications/freepublications/publication/key-world-energy-statistics.html). Therefore, the world electricity demand is at least 1.4 times of present-day electricity consumption if all the country have a HDI of 0.9.

6 Problem 6 – Electricity Energy Demand

In the International Energy Outlook 2016 (IEO2016) Reference case, world net electricity generation increases 69% by 2040, from 21.6 trillion kWh in 2012 to 25.8 trillion kWh in 2020 and 36.5 trillion kWh in 2040. (http://www.eia.gov/outlooks/ieo/electricity.cfm) This amount of electricity generation would be sufficient to allow 4000 kWh per capita on average for current human population.

7 Problem 7 – Using Gapminder

The data show that, from 1975 to 2011, as the access to energy (e.g. energy use per person) is increased, the literacy rates, the income per person and life expectancy tend to increase, while the portion of GDP from agriculture decreases. On the other hand, CO_2 emissions per capita increase as the energy use per person is increased. It might be concluded that the increase in energy access, which results in the increased CO_2 emissions per capita, increases economic activities other than agriculture that reduce the percent of GDP from agriculture and increase the income per person, and thus can improve the quality of life.