Solution to Homework 2

January 28, 2017

Problem 1 – Temperature inside a Box

For a box in heat balance with its surroundings, the power it emits should be equal to the power it absorbs,

$$P_{\text{ab}} = P_{\text{em}}.$$

In this case, the power can be reduced to power flux, j, as the surface area is constant.

The power flux absorbed by the box is written as

$$j_{\text{ab}} = (1 - R)I_0.$$

Since the wall is a perfect black body, according to Stefan–Boltzmann law, it would emit a power flux of

$$j_{\text{wall}} = \sigma T^4,$$

where T is the temperature of the box and $\sigma = 5.67 \times 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4}$ is the Stefan–Boltzmann constant. Due to the screening effect of the window, only a fraction (β) of the emitting energy escapes from the box, thus emitting power flux is

$$j_{\text{em}} = \beta j_{\text{wall}} = \beta\sigma T^4.$$

Combining these equations, we can obtain the expression for box temperature,

$$T_{\text{eq}} = \left[\frac{(1 - R)I_0}{\beta\sigma}\right]^{\frac{1}{4}}. \quad \square$$

Problem 2 – IR Radiation Absorption

From a depth of x to $x + dx$, the change in IR radiation intensity, $dI(x)$, is proportional to the change in intensity is proportional to the density of greenhouse gases $N_{1,2}$, the absorption cross-section $\sigma_{1,2}$ and the absorption probability $s_{1,2}$, i.e.

$$dI(x) = -\left(s_1\sigma_1 N_1 + s_2\sigma_2 N_2\right) I(x)\, dx.$$

Upon integration from 0 to d gives the expression of the IR radiation intensity as a function of the thickness d,

$$\begin{aligned}
\int_{I(0)}^{I(d)} \frac{dI}{I(x)} &= -\int_0^d (s_1\sigma_1 N_1 + s_2\sigma_2 N_2)\ dx, \\
\ln I(x)\Big|_0^d &= -(s_1\sigma_1 N_1 + s_2\sigma_2 N_2)\, d, \\
\ln \frac{I(d)}{I(0)} &= -(s_1\sigma_1 N_1 + s_2\sigma_2 N_2)\, d.
\end{aligned}$$

Therefore, the transmission coefficient for IR radiation, β, is written as

$$\beta \equiv \frac{I(d)}{I(0)} = \exp\left[-(s_1\sigma_1 N_1 + s_2\sigma_2 N_2)\, d\right]. \quad \square$$

Problem 3

Infra-Red Emission of Atmosphere and Earth

Assume the atmosphere is a single layer. Both the earth's surface and the atmosphere emit infra-red (IR) radiation. If the atmosphere emits an IR flux, E_a, some of it goes upward into space and the rest goes downward to the earth. A typical split is $f_{\text{space}} = 36\%$ to space and $f_{\text{earth}} = 64\%$ to the earth. Then the IR emission from earth's surface, E_s, is balanced with part of IR emissions from atmosphere in addition to the influx from the visible light, i.e.

$$E_s = 0.64 E_a + (1 - \alpha_2)\, \beta_{\text{vis}} I,$$

where $\beta_{\text{vis}} \approx 0.49$ denotes the transmission coefficient of visible light in atmosphere and $\alpha_2 \approx 0.04$ is the reflection coefficient of the earth's surface.

On the other hand, the IR emissions from the atmosphere, E_a, is balanced with absorbed power flux from IR emissions of earth's surface, $(1-\beta_{\text{IR}})E_s$, and from the visible light, $A_1 = [1 - \alpha_1 - \beta_{\text{vis}} + \beta_{\text{vis}}\alpha_2 (1 - \beta_{\text{vis}})]\, I$, i.e.

$$E_a = (1 - \beta_{\text{IR}})\, E_s + A_1.$$

Combining these two equations and substituting the parameters given in the lecture notes, we have

$$\begin{aligned}
E_s &= \frac{318.199}{\beta_{\text{IR}} + 0.5625}, \\
E_a &= -\frac{252.84\,(\beta_{\text{IR}} - 1.4039)}{\beta_{\text{IR}} + 0.5625}.
\end{aligned}$$

When the β_{IR} varies from 0.05 to 0, the relation between the IR emissions and the IR transmission coefficient is plotted in Fig. 1. It indicates that the reduction in transmission coefficient, which may result from inclusion of the greenhouse effect, increases the IR emissions of atmosphere and earth's surface.

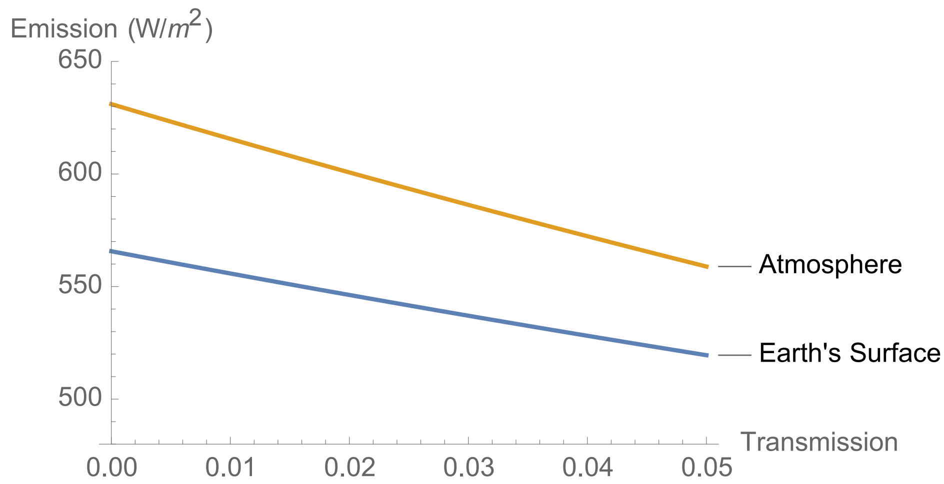

Figure 1: IR emissions of the earth's surface and the atmosphere versus IR transmission coefficient.

Temperature vs IR Transmission Coefficient

Using Stefan–Boltzmann law we can determine the temperature of the earth's surface and the atmosphere,

$$T_s = \left(\frac{E_s}{\sigma}\right)^{1/4} = 273.703\left(\frac{1}{\beta_{\mathrm{IR}} + 0.5625}\right)^{1/4},$$
$$T_a = \left(\frac{E_a}{\sigma}\right)^{1/4} = 258.414\left(\frac{1.4039 - \beta_{\mathrm{IR}}}{\beta_{\mathrm{IR}} + 0.5625}\right)^{1/4}.$$

As β_{IR} varies from 0.05 to 0, the relation between the temperature and the IR transmission coefficient is plotted in Fig. 2. It suggests that the increase in greenhouse gases emissions, which lowers the transmission coefficient, can significantly increase the temperature of atmosphere and earth's surface.

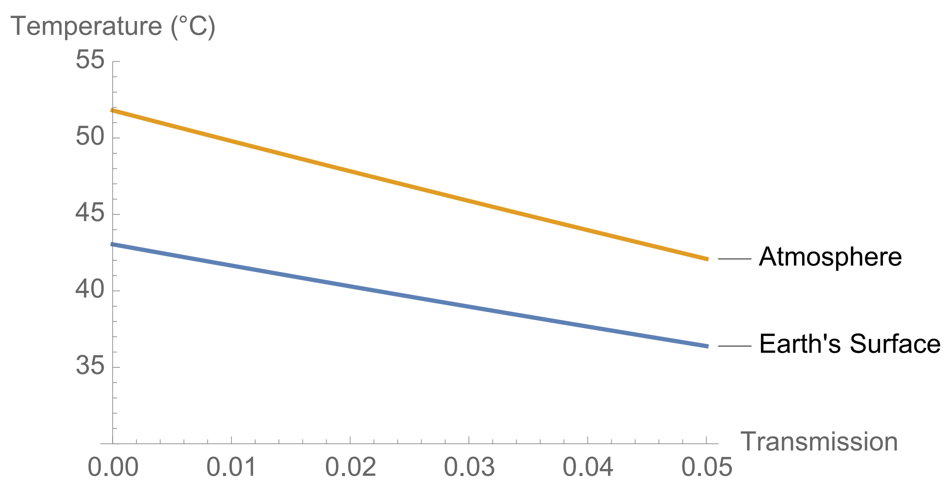

Figure 2: Temperature of the earth's surface and the atmosphere versus IR transmission coefficient.

Problem 4

As we learn from the lecture,

$$\beta \equiv \frac{I(d)}{I(0)} = \exp\left(-\sigma_{gg} n_{gg} d\right).$$

If the concentration of CO_2 is double, the transmission coefficient becomes

$$\beta' = \exp\left(-2\sigma_{gg} n_{gg} d\right) = \beta^2. \quad \square$$

Problem 5

If the CO_2 concentration is doubled, according to what we discussed in Problem 4, the transmission coefficient changes from present value 0.05 to 0.0025. Using the equations developed in Problem 3(b), we can then infer that the temperature of earth's surface will increase from 36.2 °C to 42.5 °C,

Problem 6

From Problem 1 we know the temperature can be expressed as

$$T = \left[\frac{(1-\alpha)\, I_0}{\beta\sigma}\right]^{\frac{1}{4}},$$

where $\beta = 1$ due to the absence of the atmosphere and hydrological cycle and $\alpha = \alpha(T)$ as presented in the problem.

- If $T \leq 250$ K and $\alpha = 0.5$, then the calculation shows the $T_{eq} = 234.7$ K or −38.5 °C
- If $250 \leq T \leq 270$ K and $\alpha = \frac{270-T}{40} \Rightarrow T_1 = 260.24$ K and $T_2 = 401.744$ K. Only first root suits the condition. Thus we have $T_{eq} = T_1 = 260.2$ K or −13 °C
- If $T \geq 270$ K and $\alpha = 0 \Rightarrow T_{eq} = 279.09$ K or 6 °C

The existence of those three situations indicates three meta-stable states of the system with respect to temperature perturbations. For example, initially the temperature is below 250 K and $\alpha = 0.5$, leading to a equilibrium temperature of $T_{eq} = 234.7$ K or −38.5 °C. If there is an increase in temperature from the initial value to a value between 250 and 270 K, the albedo increases and the equilibrium temperature would increase to $T_{eq} = 260.2$ K. As the temperature keeps increasing and exceeds 270 K, the albedo becomes zero $\alpha = 0$ and an equilibrium temperature of $T_{eq} = 279.09$ K or 6 °C will be reached. The opposite process can also occur. As the temperature keeps decreasing from $T > 270$ K, the α would decreases and temperature would stabilize at $T_{eq} = 234.7$ K once it drops below 250 K where α is fixed at 0.5.

Problem 7

- The ERoEI value of conventional oil/gas was about 100 in 1930s (from shallow land-based wells) and has shrunk to $20-30$ in 2000s.[1]
- The ERoEI value of fracked oil/gas is about $10-20$. Some studies suggest that the oil produced by the hydraulic-fracturing technique has an ERoRI similar to that of US conventional oil. But due to the lower quality of fracked oil, the refining cost will increase significantly.
- Energy payback time (EPBT) of the solar PV system is defined as

$$\begin{aligned} \text{EPBT} &= \frac{\text{Embedded energy}}{\text{Annual energy generated by the system}}, \\ &= \frac{W_1}{\epsilon I}, \\ &\approx 3\,(\text{year}), \end{aligned}$$

  where $W_1 \approx 600\ \text{kW h m}^{-2}$ denotes the energy required to produce unit square meter solar PV system, $I \approx 1700\ \text{kW h m}^{-2}$ is the averaged sunlight energy per year, and $\epsilon \approx 0.12$ the standard conversion efficiency.[2] The ERoEI value is hence about 10 for a solar PV system that has a lifetime of 30 years.

[1]Reference see http://dx.doi.org/10.1016/j.enpol.2013.05.049
[2]Reference see http://dx.doi.org/10.1016/j.rser.2015.02.057