MAE 119 Homework 5
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Quiz 5 to be on Monday 5 March and will **cover wind power**

1. A wind turbine is designed with a 100 m diameter rotor, and can operate in a maximum wind-speed of 10 m/sec (i.e. the turbine will "feather" the blades so that no power is produced if the wind speed exceeds this value). What is the maximum possible power output of the turbine?

2. You have been asked to do a conceptual design layout of a wind farm intended to produce a maximum of 1000 MW of power. If the turbine described in problem 1 is the one to be considered for this wind farm, and you can extract a maximum of 1 MW of power per km^2 of land area (due to turbine-turbine interactions as well as effects of the turbines slowing down the wind), what is the spacing between turbines? How many turbines are required to meet the maximum design power? If the capacity factor of the site is 0.35, what will be the average power? Research the capital costs of wind turbines. What would this wind farm cost to install?

3. A wind turbine with 100 m diameter rotor has a wind cut-in speed of 3 m/sec and a cut-out speed of 10 m/sec. It is placed in a region where the wind speed has a uniform probability, f=0.1 for wind speed $0 < V < 10$ m/sec, and f=0 for $V>10$ m/sec. What is the average power that this turbine produces? What is the maximum power it can produce (sometimes referred to as the rated power)? What is the capacity factor for this turbine?

4. Estimate the maximum wind power potential of the U.S. if 20% of the U.S. sites with wind power density exceeding 300 W/m^2 are used, and we are willing to suffer a 10% reduction in wind speed due to the placement of large arrays of wind turbines across the country. If all current U.S. electrical demand loads are constant, and we then replace 100 million of our automobile fleet with electric vehicles that each consume 10 kW-hr of electrical energy per day, what is the new U.S. electrical energy demand? What fraction of this demand can then be met with the wind turbine array deployment scenario outlined in problem 2?

Solution to Homework 5

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1 Wind Turbine Output Power

According to Betz's law, the maximum power of the turbine is

$$P_{\mathrm{max}} = \frac{16}{27}\frac{1}{2}\pi\rho r_0^2 v^3.$$

Given that the density of air is about $1\,\mathrm{kg/m^3}$, radius of rotor is $r_0 = 50\,\mathrm{m}$ and maximum speed is $v = 10\,\mathrm{m/s}$, the resulting maximum power of the wind turbine is about 2.3 MW.

2 Wind Farm Design

The required number of turbines is

$$N_t = \frac{P_{\mathrm{tot}}}{P_t} \approx \frac{1000}{2} = 500.$$

The total land area of the wind farm is

$$A_{\mathrm{tot}} = \frac{1000\,\mathrm{MW}}{1\,\mathrm{MW/km^2}} = 1000\,\mathrm{km}^2.$$

The spacing between turbines is $A_t = A_{\mathrm{tot}}/N_t = 2\,\mathrm{km}^2$.

The average power produced is

$$P_{\mathrm{ave}} = C_{\mathrm{wp}} P_{\mathrm{max}} = 0.35 \times 1000\,\mathrm{MW} = 350\,\mathrm{MW}.$$

The install cost is roughly 1.7×10^6 \$/MW.[1] The cost of this wind farm would be 1.7 billion dollars.

[1] http://www.awea.org/falling-wind-energy-costs

3 Power Distribution

The wind power is $P = \frac{1}{2}\frac{\pi}{4}d_0^2\rho v^3$. The average power output from the given speed distribution function, $f(v)$, is

$$P_{\text{ave}} = \frac{\int_3^{10} \frac{\pi d_0^2}{8}\rho v^3 f(v)\,dv}{\int_0^{10} f(v)\,dv}, \\ \approx 1\,\text{MW}.$$

To achieve the maximum possible power, the turbine is running at maximum wind speed $v = 10\,\text{m/s}$. Therefore, maximum power is

$$P_{\max} = \frac{16}{27}\frac{\pi}{8}d_0^2\rho v_{\max}^3 \approx 2.5\,\text{MW}.$$

The corresponding capacity factor is

$$C_{\text{wp}} \approx 40\%.$$

4 Wind Power Potential

Since 20% of the land $6 \times 10^5\,\text{km}^2$ are used, $A_{\text{tot}} = 1.2 \times 10^5\,\text{km}^2$. The reduction in wind speed is $\frac{\delta u}{u} = -0.1$, which leads to a loss in wind power density is $\frac{\delta P}{P} = -0.3$. The perturbation equation yields

$$\frac{\delta u}{u} = -\frac{2.5}{4}\frac{u}{w}\left(1 + \frac{H}{h}\right)\lambda,$$

where $u = 10\,\text{m/s}$, $w = 0.5\,\text{m/s}$, $h = 50\,\text{m}$ and $H = 400\,\text{m}$. Therefore, we have $\lambda \approx 2.5 \times 10^{-3}$. The total available wind power is thus

$$A_{\text{tot}}\lambda P \approx 1.2 \times 10^5\,\text{m}^2 \times 1 \times 10^6 \times 2.5 \times 10^{-3} \times 300 = 9 \times 10^{10}\,\text{W}.$$

The increase in electricity demand per day due to electric vehicles is

$$10 \times 10^3 \times 3600 \times 100 \times 10^6 = 3.6 \times 10^{15}\,\text{J}.$$

The wind energy available per day is about $9 \times 10^{10} \times 3600 \times 24 = 7.76 \times 10^{15}\,\text{J}$. By considering the conversion efficiency and the capacity factor, the wind turbine array sketched in Problem 2 produces a power of $0.35 \times 0.59 \times 7.76 \times 10^{15} = 1.6 \times 10^{15}\,\text{J}$, which is about 40% of the increased energy demand per day due to the increased electrical vehicles.