Homework 6 Solar PV Energy
MAE 119 W2017
Professor G.R. Tynan

1. What is the most likely wavelength and frequency of light emitted from the sun which has a black body temperature of about 6600 deg K? What photon energy does this correspond to?

2. Plot the blackbody spectrum of emission from the sun, and identify the photon wavelength or frequency that corresponds to the bandgap energy of Si. Identify which region has photons with enough energy to create charge carrier pairs in a SI based solar cell.

3. 2. If the bandgap energy of Si is 1.1 eV, estimate the intrinsic or maximum efficiency for a Si-based solar cell exposed to the sun's blackbody emission spectrum.

4. 3. A typical Si-based PV cell will have a form factor, FF=0.7, and an open circuit voltage V_{OC}=0.6 V and a short-circuit current density J_{SC}=30 mA/cm^2. What is the maximum real efficiency of this cell? Suppose now that by improving the manufacturing process, you are able to increase the carrier lifetime by a factor of 3. Since the open-circuit voltage doesn't depend on defect density, what will be the new short-circuit current density if nothing else is changed in the PV cell design? By how much will the efficiency improve?

5. Estimate the solar PV system area and electrical energy storage required to power the UCSD campus purely by sunlight. If a solar PV system with a capacity factor of 15% can be installed for $3/Watt, and the energy storage costs are $200/kW-hr, what would be the up-front installation cost of such a system? If it lasts for 20 years and performs to its ideal specifications year-round (i.e. you don't have to consider losses due to bad weather, clouds, fog etc) and interests costs for money are zero so that the present value and future value of money is equal (really an ideal world!)), how much does the electrical energy cost per kW-hr? How does this compare against conventional electrical energy costs?

6. The San Diego region is considering the installation of a pumped hydro storage system to smooth out the time-variation of renewable energy sources. Suppose the upper and lower storage level reservoirs both have a surface area of 3 km^2, and a uniform depth of 100m. The vertical displacement between the center-of-gravity of the two reservoirs is 300m. You have been asked to design a system to provide constant power from this system to San Diego over a 12 hour night interval, and then recharge the

system with excess solar PV power. How much power can you promise to deliver? How much energy does this correspond to? What is the water flow rate through the system? How much solar PV power do you need to divert to the pumps to recharge the system during the day?

7. Estimate the maximum energy storage capacity of a 1m diameter, 20 cm thick flywheel made of high strength steel, operating with stresses at the periphery of the flywheel that are approaching the ultimate tensile strength of the material. What is the specific energy storage (i.e. J/kG) of this device? How does this compare to batteries? To liquid fuels? *Hint:* The flywheel will fail when the tensile strength of the material at the perimeter approaches the kinetic energy density of that region. How might you modify the design of the flywheel to increase the specific energy storage capacity?

8. High temperature superconducting cables are beginning to be available for commercial use. Investigate their performace on the Internet, and estimate the energy storage capability of a solenoidal magnet constructed with these materials. Suppose the coil is 1m long, and has an inner bore of 10 cm. The coil operation will be limited by the maximum current density in the cables, the maximum allowable magnetic field, and the mechanical strength of the structure holding the coil together. [Note: this is an open ended problem … no single solution. Just want to get you thinking].

Solution to Homework 6

March 10, 2017

1 Wavelength and Frequency of Peak Emission

The blackbody spectral radiance (power per emitting area per solid angle per unit wavelength) is given by

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)}.$$

Letting $\partial_\lambda B(\lambda, T) = 0$ gives

$$B(\lambda, T)\left(\frac{hc}{\lambda^2 kT}\frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1} - \frac{5}{\lambda}\right) = 0.$$

By defining $x = \frac{hc}{\lambda kT}$ the equation reduces to

$$\frac{xe^x}{e^x - 1} = 5,$$

which yields a numerical root of $x = 4.965$. At $T = 6600\,\mathrm{K}$, the most likely wavelength is $\lambda_m = \frac{hc}{xkT} = 439.6\,\mathrm{nm}$. The corresponding photon energy is about $h\nu = 2.82$ eV.

On the other hand, we have the frequency dependent expression of the blackbody spectral radiance, i.e.

$$B(\nu, T) = \frac{2h\nu^3}{c^2 \left(e^{\frac{h\nu}{kT}} - 1\right)}.$$

Similarly, letting $\partial_\nu B(\nu, T) = 0$ gives

$$\frac{xe^x}{e^x - 1} = 3,$$

which yields a root of $x = 2.821$. At $T = 6600\,\mathrm{K}$, the most likely frequency is $\nu_m = xkT/h = 2.88 \times 10^{14}$ Hz.

Many devices and systems respond in proportion to the number of incident photons, and it is useful to express radiometric quantities in terms of photons per second rather than watts. Details can be found in references (http://spectralcalc.com/blackbody/planck_blackbody.html).

2 Emission upon PV Cells

The blackbody emission spectra at $T = 6600$ K is shown in figure 1. The silicon has an energy gap of $E_g = 1.1$ eV, corresponding to a wavelength of $\lambda = 1127.9$ nm. Photons with a wavelength less than 1127.9 nm are expected to produce the electron-hole pairs in the p-n junction.

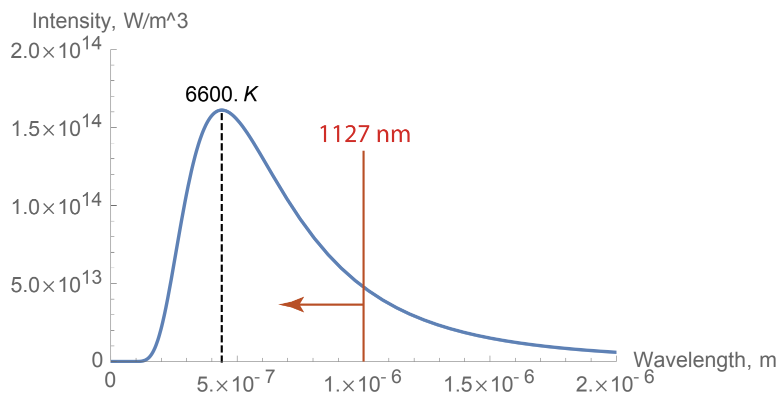

Figure 1: Blackbody emission spectra at $T = 6600$ Kelvin.

The most commonly known solar cell is configured as a large-area p-n junction made from silicon. The current is generated by the separation of electron-hole pairs that are created in the depletion zone.

3 Maximum Conversion Efficiency of PV Cells

The incident power per unit area per steradian, due to radiation at T_s falling upon the device, will be P_s, which can be obtained by integrating the Planck distribution with respect to the frequency, i.e.

$$\begin{aligned} P_s &= \frac{2h}{c^2} \int_0^\infty \frac{\nu^3}{\exp(h\nu/kT_s) - 1} \, d\nu, \\ &= \frac{2(kT_s)^4}{h^3c^2} \int_0^\infty \frac{x^3}{e^x - 1} \, dx, \\ &= 2\pi^4 \frac{(kT_s)^4}{15h^3c^2}, \end{aligned}$$

where $x \equiv h\nu/kT_s$.

On the other hand, the output power per unit area per steradian will be given by $h\nu_g Q_s$. The Q_s is the photon flux which is obtained by dividing the P_s by the photon energy $h\nu$, i.e.

$$
\begin{aligned}
Q_s &= \frac{2}{c^2}\int_{\nu_g}^{\infty} \frac{\nu^2}{\exp(h\nu/kT_s)-1}\,d\nu, \\
&= \frac{2(kT_s)^3}{h^3c^2}\int_{x_g}^{\infty} \frac{x^2}{e^x-1}\,dx,
\end{aligned}
$$

where $x_g \equiv h\nu_g/kT_s$.

According to the above definitions, the maximum conversion efficiency is expressed as

$$
\begin{aligned}
\eta &= \frac{h\nu_g Q_s}{P_s}, \\
&= \frac{x_g \int_{x_g}^{\infty} \frac{x^2}{e^x-1}\,dx}{\pi^4/15}.
\end{aligned}
$$

For Si the energy gap is $E_g = 1.1$ eV, and the corresponding $x_g = E_g/kT_s$ where $T_s = 6000$ K. The resulting efficiency is approximately 44%.

4 Real Efficiency of PV Cells

By considering the fill factor, the maximum real efficiency will be given by

$$
\eta = \mathrm{FF} \times \frac{V_{\mathrm{oc}} j_{\mathrm{sc}}}{I_0}.
$$

The I_0 is the solar irradiance on earth surface which is approximately $1000\,\mathrm{W\,m^{-2}}$. The resulting real efficiency is about $\frac{0.7*0.6*300}{1000} = 12.6\%$.

The short circuit current density is proportional to square root of the carrier lifetime, i.e. $j_{\mathrm{sc}} \propto L_{e,h} \propto \tau_{e,h}^{1/2}$, therefore $j'_{\mathrm{sc}} = \sqrt{3} j_{\mathrm{sc}}$ if the carrier lifetime is increased by a factor of 3. The resultant efficiency is then $\eta' \approx 21.8\%$.

5 PV System for UCSD Campus

The average electricity power usage of whole UCSD campus is roughly $P_c = 10\,\mathrm{MW}$. The daily electrical energy consumed in UCSD campus is then $E_c = 240\,\mathrm{MW\,h}$.

The solar PV system with a capacity factor of 15% should be able to produce a peak power of $P_{\mathrm{pv}} = P_c/0.15 \times 24/12 \approx 133\,\mathrm{MW}$ (assume the PV system only operates 12 hours a day). The corresponding cost of the PV system is $133 \times 10^6 \times 3 = 4 \times 10^8$ \$.

The energy storage system will cost $0.5E_c \times 1000 \times 200 = 0.24 \times 10^8$ \$. Here we assume the energy storage system can support a 12-hour (overnight) operation for the UCSD campus.

If the whole system runs for 20 years, it will produce a total electrical energy produced of $E_{\text{tot}} = E_c \times 365 \times 20 = 1.752 \times 10^9$ kW h. The cost of electricity will be $\frac{4.24 \times 10^8}{1.752 \times 10^9} = 0.24\,\$/(\text{kWh})$, which is about twice of the current price of conventional electricity.

6 Pumped Hydro-Power

The maximum volume of the water that can be stored in each reservoir is about

$$V = 3 \times 10^6 \times 100 = 3 \times 10^8\,\text{m}^3.$$

The total potential energy of the water is then

$$U_{\text{tot}} = \rho V g H = 9 \times 10^{14}\,\text{J}.$$

If the water is released from upper reservoir to the lower one in a duration of 12 hours, the delivered power is

$$P = U_{\text{tot}}/(12 \times 3600) = 2.1 \times 10^{10}\,\text{W}.$$

The volumetric flow rate of the water is

$$V/(12 \times 3600) \approx 6.9 \times 10^5\,\text{m}^3\,\text{s}^{-1}.$$

Assuming that the PV system works for 10 hours a day, the required PV power should be 25 GW.

7 Flywheels

The stress of the steel is about $\sigma = 900$ MPa. The density of the steel is $\rho = 8 \times 10^3\,\text{kg}\,\text{m}^{-3}$. The specific energy of the flywheel can be expressed as

$$\frac{E}{m} = K\frac{\sigma}{\rho} \approx 100\,\text{kJ}\,\text{kg}^{-1}.$$

The total stored energy is about $E = 100 \times 10^3 \times \rho V = 1.26 \times 10^8$ J.

The Lithium-ion battery has a specific energy of roughly $500\,\text{kJ}\,\text{kg}^{-1}$. The specific energy of the fuel oil is about $46\,\text{MJ}\,\text{kg}^{-1}$.

The specific energy of the flywheels can be improved by using low density high strength materials like fiber carbon.

8 Superconducting Magnetic Energy Storage

The magnetic energy stored by a coil carrying a current is given by

$$E = \frac{1}{2} L I^2,$$

where L is the inductance and I is the current. Given that the coil (solenoid) is $l = 1$ m long and has an inner bore of $R = 10$ cm with $N = 200$ turns, the inductance of the coil (solenoid) can be estimated as

$$L = \frac{\mu N^2 \pi R^2}{l} \approx 0.08\,\mathrm{H},$$

where the relative permeability of the core material is assumed to be about $\mu/\mu_0 \approx 200$. For classic superconductors, the critical current density is about $j_c = 10^4\ \mathrm{A\,cm^{-2}}$. Therefore, the resulting maximum magnetic energy stored in the superconducting coil is $E = 4 \times 10^6\,\mathrm{J\,cm^{-2}}$.