MAE 119 QUIZ 2
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Students may use any reference materials.

We consider the heat balance of a coupled atmosphere-Earth subjected to visible light input from the sun. The atmosphere is a uniform slab as we have discussed in our lectures with a thickness d. The atmosphere contains an infra-red absorbing gas with a constant absorption cross-section given as σ. The density of this gas is varying in time according to the equation

$$n_{gg}(t) = n_0(1+\alpha t)$$

where n_0 denotes the density at t=0 and α is the rate (which is constant) at which the greenhouse gas density is increasing.

The infra-red emissivity from the atmosphere and Earth surface are given as E_1 and E_2 and are given as

$$E_1 = A_1 + (1-\beta_{IR})E_2$$
$$E_2 = A_2 + f_{Earth}E_1 \, .$$

In this quiz just treat A_1, A_2 and f_{Earth} as known constants and don't worry about the values of the albedo, visible light transmission coefficient, or solar intensity. Thus you may express your answers below in terms of these quantities.

a) What is the infra-red transmission coefficient and how does it change in time, i.e. what is $\beta_{IR}(t)$?
b) How do the infra-red emissivities E_1 and E_2 change in time?
c) Using what you know about blackbody radiation, how does the temperature of the atmosphere and Earth surface change in time?
d) What will be the final temperature of the atmosphere and Earth when time $t=1/\alpha$.

SOLUTION:

a) we know that $\beta_{IR} = \exp\left[-n\sigma d\right]$. Since we are given n=n(t), we can then simply write $\beta_{IR} = \exp\left[-n_0\left(1+\alpha t\right)\sigma d\right]$. 5 POINTS.
b) Solving the two given equations for E1 and E2 gives

$$
\begin{aligned}
E_1 &= A_1 + (1-\beta_{IR})E_2 \\
&= A_1 + (1-\beta_{IR})\left(A_2 + f_{Earth}E_1\right) \\
&= A_1 + A_2(1-\beta_{IR}) + f_{Earth}E_1(1-\beta_{IR})
\end{aligned}
$$

or

$$E_1 - f_{Earth}E_1(1-\beta_{IR}) = A_1 + A_2(1-\beta_{IR})$$

or

$$E_1\left[1 - f_{Earth}(1-\beta_{IR})\right] = A_1 + A_2(1-\beta_{IR})$$

or

$$E_1 = \frac{A_1 + A_2(1-\beta_{IR})}{1 - f_{Earth}(1-\beta_{IR})}$$

This then gives

$$E_2 = A_2 + f_{Earth}\frac{A_1 + A_2(1-\beta_{IR})}{1 - f_{Earth}(1-\beta_{IR})}$$

5 POINTS

c) the equilibrium temperature is related to the emissivity by the Stefan-Boltzmann relation

$$T = \sqrt[4]{\frac{E}{\sigma_{bb}}}$$

We can use the expressions from part (b) to find the atmosphere and Earth surface temperature, T1 and T2. 5 POINTS.

d) taking t=1/α gives $\beta_{IR} = \exp\left[-2n_0\sigma d\right] = \beta_{IR}^2\big|_{t=0}$ which can then be inserted into the expressions for E1 and E2 in part (b), and then used to solve for T according to the relation in part (c). 5 POINTS.