MAE 119 Winter 2017 Quiz 2
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Closed book closed notes.

1. A planet is illuminated with a visible light intensity, I, has a surface albedo $\alpha > 0$ and has a cloud-free atmosphere that is perfectly transparent in the visible part of the spectrum. The atmosphere is a uniform slab with a thickness d and contains an infra-red (IR) absorbing gas of density n molecules/unit volume that have a constant IR absorption cross-section given as σ, resulting in an IR transmission coefficient, $0 < \beta < 1$. As a result, the atmosphere warms to a finite temperature and then re-radiates 50% this energy towards the planets surface and 50% to space.
    a. Derive an expression for the infrared transmission coefficient β in terms of the absorbing gas density n, the atmosphere thickness d, and the absorption cross-section σ. 10 points.
    b. Using a simple energy balance analysis as done in class, find the infra-red emissivity E_{surf} of the planet's surface and E_{atm} of the atmosphere. 10 points
    c. What are the corresponding surface and atmospheric temperatures if they both act as ideal blackbodies? *Hint:* the radiation emission/unit area, E, from an ideal blackbody is given as $E = \sigma T^4$ where σ denotes the Stefan-Boltzmann constant and T is the temperature in absolute units. 10 points
    d. If the concentration of IR-absorbing molecules in the atmosphere were to double, what are the new values of these two temperatures? 10 points
    e. If these new temperatures were to result in an elimination of all visible light reflection at the surface, what happens to the albedo? What effect does this then have on the surface and atmospheric temperatures? 10 points

2. Derive the ideal heat engine (i.e. Carnot) efficiency. [Remember that the heat transferred from the hot reservoir into the engine is given as, q_h, the hot reservoir temperature is held at T_h and the heat rejected to the cold reservoir is q_c and the cold reservoir sits at a temperature T_c.]
    a. Draw a control volume surrounding the engine and label the heat input, the heat rejected, the two reservoirs, and the work, w, done by the engine on the environment. 10 points.
    b. Using the principle of conservation of energy, write down an expression for the work, w, in terms of q_h and q_c. 10 points.
    c. Knowing that such an engine satisfies the relation $\frac{q_c}{T_c} = \frac{q_h}{T_h}$,

       Find the efficiency of the engine. 10 points.

Solution to Quiz 2

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Problem 1

a.

The relative change of IR intensity is proportional to gas density n and absorption cross-section σ, i.e.

$$dI(x) = -\sigma n I(x) dx.$$

Upon integration from 0 to d gives

$$\int_{I(0)}^{I(d)} \frac{dI}{I(x)} = -\int_0^d (\sigma n)\, dx,$$
$$\ln I(x)\Big|_0^d = -(\sigma n) d,$$
$$\beta \equiv \frac{I(d)}{I(0)} = \exp[-(\sigma n) d].$$

b.

Since the atmosphere albedo $\alpha_1 = 0$, visible light transparency $\beta_{\text{vis}} = 1$ and $f_{\text{earth}} = 0.5$, the 0-D energy balance model is reduced to

$$E_s = (1-\alpha)I + f_{\text{earth}} E_a, \tag{1}$$

$$E_a = (1-\beta)E_s, \tag{2}$$

where $\beta = \beta_{\text{IR}}$ is the transmission coefficient of infra-red lights. The solution is then

$$E_s = \frac{(1-\alpha)I}{1 - f_{\text{earth}}(1-\beta)} = \frac{2(1-\alpha)}{(1+\beta)}I, \tag{3}$$

$$E_a = \frac{(1-\beta)(1-\alpha)I}{1 - f_{\text{earth}}(1-\beta)} = \frac{2(1-\beta)(1-\alpha)}{(1+\beta)}I. \tag{4}$$

c.

Using Stefan-Boltzmann law, we can determine the temperature expression,

$$T_s = \sqrt[4]{\frac{2(1-\alpha)}{(1+\beta)\sigma_{\text{SB}}}I}, \tag{5}$$

$$T_a = \sqrt[4]{\frac{2(1-\beta)(1-\alpha)}{(1+\beta)\sigma_{\text{SB}}}I}. \tag{6}$$

d.

When the concentration of IR-absorbing molecules in the atmosphere is doubled, the IR transmission coefficient becomes β^2. The temperature expression is then

$$T_s = \sqrt[4]{\frac{2(1-\alpha)}{(1+\beta^2)\sigma_{\text{SB}}}I}, \tag{7}$$

$$T_a = \sqrt[4]{\frac{2(1-\beta^2)(1-\alpha)}{(1+\beta^2)\sigma_{\text{SB}}}I}. \tag{8}$$

The decrease in the IR transmission coefficient would increase the temperature of both the Earth's surface and the atmosphere.

e.

If these new temperatures were to result in an elimination of all visible light reflection at the surface, the albedo becomes zero, $\alpha = 0$. New temperature becomes

$$T_s = \sqrt[4]{\frac{2}{(1+\beta)\sigma_{\text{SB}}}I}, \tag{9}$$

$$T_a = \sqrt[4]{\frac{2(1-\beta)}{(1+\beta)\sigma_{\text{SB}}}I}. \tag{10}$$

The expression indicates the elimination of visible reflection at the surface would further increase the temperature of both the Earth's surface and the atmosphere.

Problem 2

a.

The heat engine diagram is shown in Fig. 1.

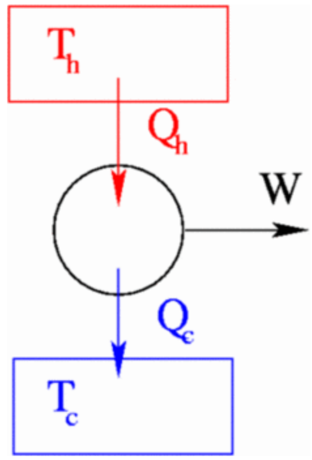

Figure 1: Heat engine diagram.

b.

Referring to figure 1, the amount of work done over a cycle for a reversible process is

$$W = \oint (dQ - dU) = \oint dQ = q_h - q_c.$$

c.

The efficiency of a heat engine is defined as

$$\eta = \frac{W}{q_h} = 1 - \frac{q_c}{q_h}.$$

The Carnot heat engine satisfies the relation $\frac{q_c}{T_c} = \frac{q_h}{T_h}$, since the entropy remains unchanged over a Carnot cycle. The efficiency is then written as

$$\eta = 1 - \frac{T_c}{T_h}.$$