MAE 119 Professor G.R. Tynan
Winter 2015
Quiz 3 Closed Book/Closed Notes

In our simple carbon balance model, we assumed that the flux of C between the atmosphere and land & ocean was proportional to the deviation of carbon concentration from the equilibrium value. The atmospheric carbon balance model could then be written as

$$\frac{\partial}{\partial t}\delta M_C(t) = Q_C(t) - \frac{\delta M_C}{t_{net}}.$$

Where t_{net} denotes the effective timescale for atmospheric C exchange with the Earth's surface and oceans, $\delta M_C(t)$ denotes the deviation of the atmospheric carbon content away from the equilibrium value, and $Q_C(t)$ is the carbon injection rate from fossil fuel combustion.

Suppose the carbon source, Q_C=0 for time t < 0, and then at t=0 the carbon source injection rate instantaneously increases to Q_C =10 GigaTonnes/year and is held constant. The effective absorption timescale is given as $t_{net} = 1000\,years$.

a) For time t < 0 what is δM_C? 10 POINTS
b) Just after the source is turned on (i.e. 0< t << 1000 years) δM_C hasn't changed much from the t<0 value. In this case, the differential equation for δM_C can be simplified by neglecting a term. What does the differential equation look like? 10 POINTS
c) What is the approximate functional form for the growth of δM_C for 0< t << 1000 years, i.e. shortly after the source is turned on ? 10 POINTS
d) Sketch the time evolution of δM_C for t approaching 1000 years and longer. How long will it take for δM_C to get within about 70% of its final value after the change in injection rate at t=0? One significant figure will suffice. 10 POINTS

/40

MAE 119
Quiz 3 Solution
W2015

/10 a) $Q_{C_0} = \begin{cases} 0 & t<0 \\ 10 & t>0 \end{cases}$

$$\delta M_c = Q_{C_0} t_{net} (1 - e^{-t/t_{net}})$$

$t<0$ $\quad Q_{C_0} = 0 \rightarrow \boxed{\delta M_c = 0}$ $\quad$ +10

/10 b)

$$\frac{d}{dt} \delta M_c(t) = Q_c(t) - \frac{\delta M_c}{t_{net}}$$

δM_c for $t<0 = 0$

if δM_c doesn't change much initially for $t>0$ but $t \ll 1000$,

$$\delta M_c \big|_{t<0} \simeq \delta M_c \big|_{\substack{t>0 \\ t \ll 1000}} \simeq 0$$

$$\boxed{\frac{d}{dt} \delta M_c(t) = Q_c(t)}$$

+10

for $t > 0$ $\quad Q_c(t) = Q_{c0} = 10$ GTonnes/yr

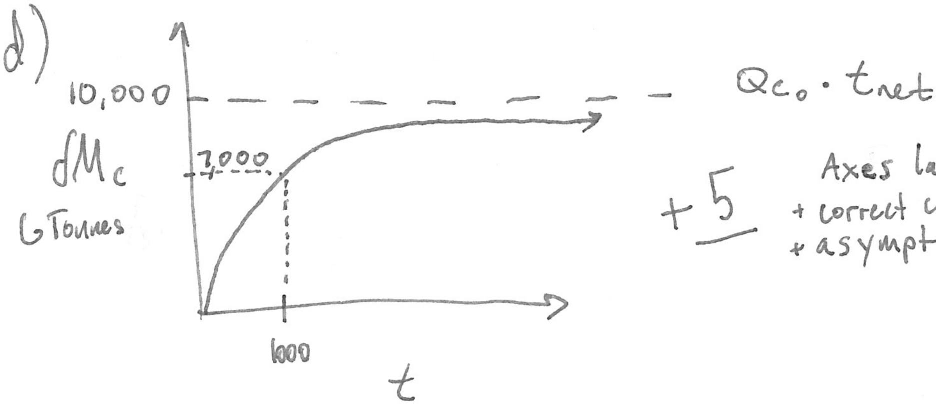
/10 c) $\quad \frac{d}{dt}\,\delta M_c(t) = Q_{c0}$

$$\int_0^t \left[\frac{d}{dt}\,\delta M_c(t) = Q_{c0} \right] \rightarrow \underline{\delta M_c(t) = Q_{c0}\, t}$$

which is $\boxed{\text{\underline{linear} with time } t.}$ $\quad \underline{+10}$

qualitative answer is sufficient

/10 d)



$\underline{+5}$ Axes labled + correct curve + asymptote @ 10,000

$$\delta M_c(t) = Q_{c0}\, t_{net} \left(1 - e^{-t/t_{net}}\right)$$

note: $1 - e^{-1} \approx \frac{3}{3} - \frac{1}{3} \approx \frac{2}{3} \approx \underline{0.7}$

if $\delta M_c(t)$ saturates at $Q_{c0} t_{net}$

then it will be $\approx 70\%$ of this

at $t = t_{net}$. $\rightarrow \underline{\delta M_c(t_{net}) = Q_{c0}\, t_{net}\,(0.7)}$

$\boxed{t = 1000 \text{ yrs}}$ $\quad \underline{+5}$