MAE 119 Professor G.R. Tynan
Winter 2017
Quiz 3 Closed Book/Closed Notes

In our simple carbon balance model, the deviation of the atmospheric carbon content away from the equilibrium value follows an equation given as

$$\frac{\partial}{\partial t}\delta M_C(t) = Q_C(t) - \frac{\delta M_C}{t_{net}}.$$

Where t_{net} denotes the effective timescale for atmospheric C exchange with the Earth's surface and oceans, and $Q_C(t)$ is the carbon injection rate from fossil fuel combustion.

Suppose the carbon source $Q_{C0}=0$ for time $t < 0$, and then at $t=0$ the carbon source injection rate begins to increase linearly in time, i.e. for $t>0$ we have $Q_C=Kt$, where K is a constant.

a) For time $t < 0$ what is δM_C? 10 POINTS
b) Suppose that t_{net} becomes very long (i.e. 100's to 1000's of years) compared to the timescale, t, we are interested in. How can you simply the above differential equation to account for this?
c) Under the assumption from part (a) above, shortly after the source is turned on t=0 (i.e. for $t \ll t_{net}$) what is the solution for $\delta M_C(t)$? 10 POINTS
d) If this carbon is injected into the atmosphere as an IR absorbing species, how does the IR transmission coefficient of the atmosphere behave in time for $t \ll t_{net}$? 10 POINTS

Solution to Quiz 3

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a)

$\delta M(t) = 0$ when $t < 0$.

b)

Since $t_{\text{net}} \to +\infty$, the equation for short t is reduced to

$$\frac{d}{dt}\delta M_c(t) = Kt.$$

c)

The solution is then

$$\delta M_C(t) = \frac{1}{2}Kt^2$$

d)

The evolution of IR transmission coefficient is written as

$$\begin{aligned}\beta(t) &= \beta_0 \exp(-\sigma d \delta n(t)),\\ &= \beta_0 \exp\left(-\frac{\sigma d}{m_C V}\delta M(t)\right),\\ &= \beta_0 \exp\left(\frac{\ln \beta_0}{M_0}\delta M(t)\right).\end{aligned}$$

For $t \ll t_{\text{net}}$, the solution is

$$\beta(t) = \beta_0 \exp\left(\frac{\ln \beta_0}{2M_0}Kt^2\right).$$