MAE 119 WINTER 2015
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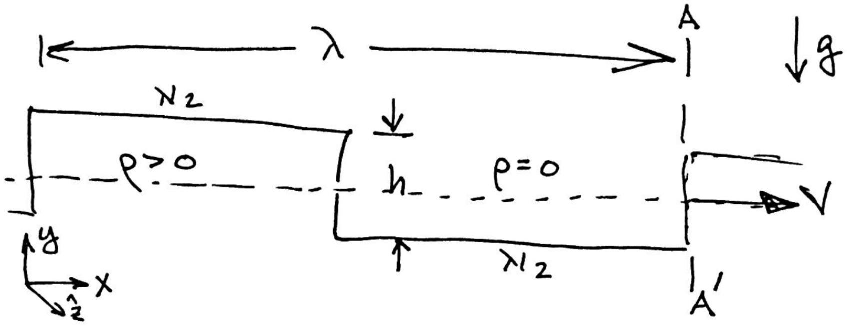
QUIZ 4 CLOSED BOOK CLOSED NOTES



1. Consider the simplified "wave" shown in the diagram above. It has a wavelength λ, height h, and propagates with a speed V, and has a period T=λ/V. (10 POINTS EACH, 30 POINTS TOTAL)

    a. For a unit depth wave, how much gravitational potential energy does this disturbance have for one wavelength λ?

    b. In one period, T, how much total wave energy passes though the plane labeled A-A' in the figure above? *Note: Assume there is an amount of kinetic energy associated with the periodic up/down motion of the fluid's surface and that this energy is equal to the gravitational potential energy found in part a.*

    c. If all of this energy is extracted by an unspecified mechanism, what is the average power that can be produced per unit depth in the z-direction?

2. A tidal basin has a surface area of 1 km^2 and experiences a 1 m tide amplitude with a period of 10 hours. What is the maximum possible power production, assuming perfect energy conversion efficiency? One significant figure is sufficient. (10 POINTS TOTAL)

Quiz 4 Solution

/30 1. a) for $0 < x < \lambda/2$, $y = h$

/10 for $\lambda/2 < x < \lambda$, $y = 0$

$$dU_{Pot.} = \rho g \, x y \, dy = \rho g \frac{\lambda}{2} y \, dy \quad +3$$

$$\int_0^h dU_{Pot.} = U_{Pot.} = \frac{\rho g \lambda}{2} \int_0^h y \, dy$$

$$= \frac{\rho g \lambda}{2} \left(\frac{h^2}{2} \right)$$

$$\boxed{U_{Pot.} = \frac{\rho g \lambda h^2}{4}} \quad +7$$

b) $U_{tot} = U_{Kin.} + U_{pot} = \underline{2 U_{pot.}} = \boxed{\frac{\rho g \lambda h^2}{2}}$ +10

/10 $U_{tot.}$ passes through the plane A-A' in a time T,

c) then max power $\boxed{P_{max} \leq \frac{U_{tot.}}{T} = \frac{\rho g \lambda h^2}{2T}}$ +10

/10

$\frac{}{10}$

2. $A = 1 \times 10^6 m$ $\rho = 1000 \ Kg/m^3$

$h = 1 m$ $g = 10 \ m/s^2$

$T = 10 \ hrs$

$$T = 10 \ h \cdot \frac{60 \ m}{1 h} \cdot \frac{60 s}{1 m} = 36{,}000 \ s$$

$$P_{max} = \frac{\rho g A h^2}{2T}$$ + 3

$$= \frac{10^3 \cdot 10 \cdot 10^6 \cdot 1^2}{2 \cdot 4 \times 10^4}$$

$$= \frac{1}{8} \cdot 10^6 = 125000 \ W$$

$$\boxed{P_{max} = 100 \ KW}$$ + 7