**MAE 119 WINTER 2017**
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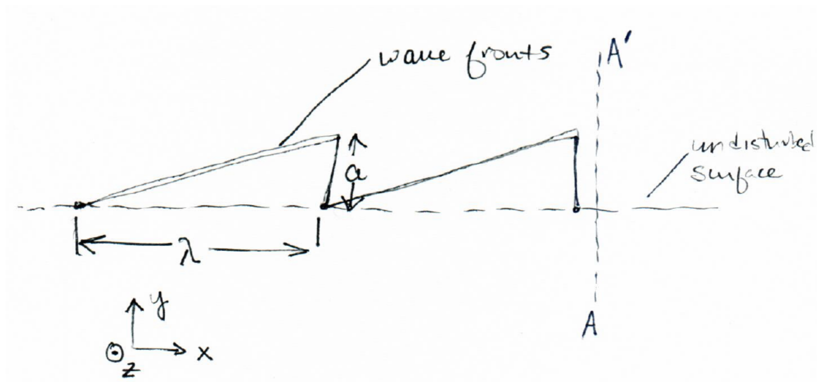
**QUIZ 4 CLOSED BOOK CLOSED NOTES**

Wave Power:




1. Consider the simplified "wave" shown in the diagram above. It has a wavelength λ, height a, and propagates to the right with a speed V, and has a period T=λ/V. What is the average power that can be produced per unit depth in the z-direction? 20 points.

Solar Thermal

2. Recall that the concentration ratio of a solar thermal power plant, C, is the ratio of the collector area to the target area. The ratio of the distance from the primary optical component to the distance from that component to the target is known as the focal number, f.
    a. Using geometric optics, *show* that the maximum acceptance angle of the power plant's optical system, $\delta\theta = 1/(f\,C)$. Hint: Remember that for specular reflection that the exit angle of light rays is equal to the entrance angle of the incident light rays and that the optics must focus the light so that it strikes the target. 10 points.
    b. If the power plant is located in a region where the atmosphere contains light-scattering particles with a cross-section, σ, and a density of n particles/volume that are uniformly distributed across an atmosphere of depth, d, what is the DNI at the ground level? The solar irradiance at the top of the atmosphere is $I_0$. 10 points.
    c. If the power plant has a collector area, A and has a heat engine with an efficiency, η, and the acceptance angle of the optics is so small that *any amount* of scattering means that the scattered radiation cannot be focused to the target, what is the power output of the power plant? 10 points.

# Solution to Quiz 4

Feb. 15, 2017

## 1 Problem 1 — Wave Power

A small fluid element $\delta m$ at the height of $y$ has a potential energy of

$$dU = \delta m\, gy = \rho dx dy\, gy.$$

In a full cycle, the total potential energy is

$$\begin{aligned} U &= \iint dU, \\ &= \rho g \int_0^{\lambda} dx \int_0^{\frac{a}{\lambda}x} y dy, \\ &= \frac{1}{2}\rho g \frac{a^2}{\lambda^2}\int_0^{\lambda} x^2 dx, \\ &= \frac{1}{6}\rho g a^2 \lambda. \end{aligned}$$

The mean kinetic energy of linear harmonic oscillators is equal to the mean potential energy, i.e. $K = U$. Taking the kinetic energy into account, the wave power is then

$$P_{tot} = \frac{U+K}{T} = \frac{1}{3}\rho g a^2 V.$$

## 2 Problem 2 — Solar Thermal Power

### 2.1 Angle Deviation

The geometry of a parabolic trough is shown in figure 1. The requirement for the focusing optics is $d_t \geqslant \delta y \approx l\delta\theta$, thus the allowable angle deviation is

$$\delta\theta \leqslant \frac{d_t}{l} = \frac{d_t}{d}\frac{d}{l} = \frac{1}{Cf}.$$

For power tower system, $\delta\theta \leqslant \dfrac{1}{C^{1/2}f}$.

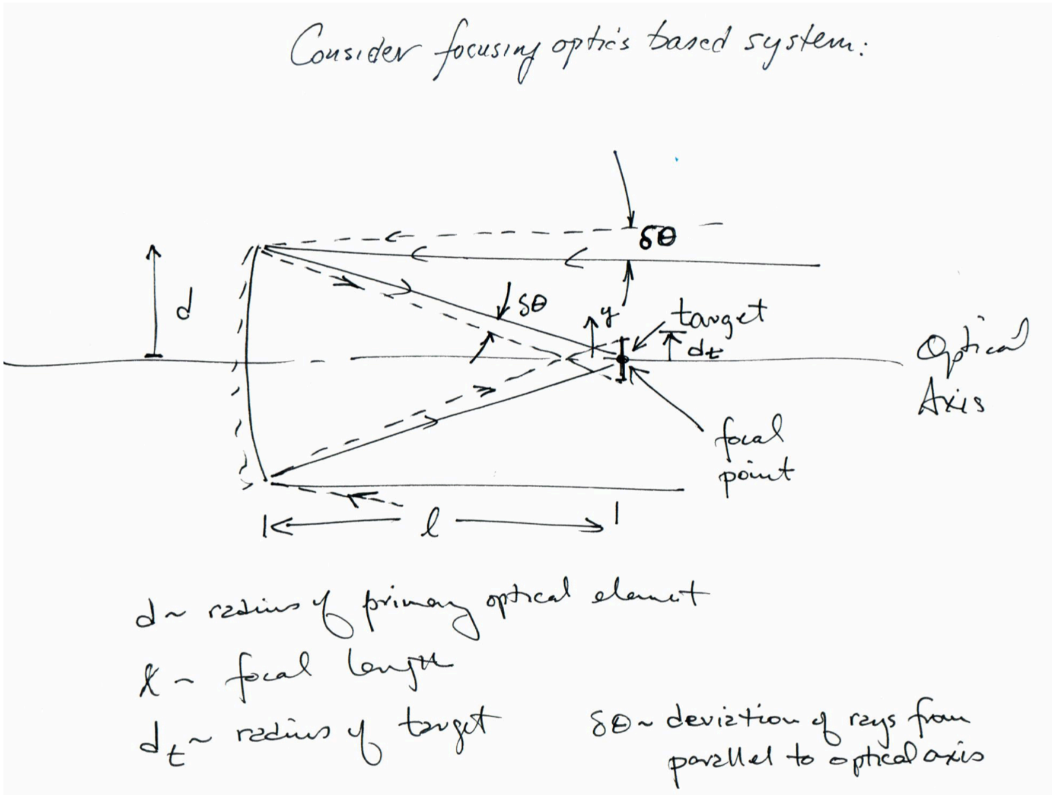



Figure 1: Geometry of a parabolic trough system.

## 2.2 DNI

$$dI = -\sigma n I_0 \, dx,$$
$$\int \frac{dI}{I_0} = -\int_0^d \sigma n \, dx,$$
$$I = I_0 \exp(-\sigma n d).$$

## 2.3 Output Power

Only DNI power input is accepted by the system, then

$$P_{in} = IA.$$

The heat engine efficiency is $\eta$, thus the output power is

$$P_{out} = \eta P_{in} = \eta IA.$$