Solution

MAE 119 WINTER 2015
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QUIZ 6 CLOSED BOOK CLOSED NOTES

At a particular site, wind blows half the time at 1 m/sec, and the other half of the time at 10 m/sec. You are considering the installation of a small wind-turbine with rotor diameter of 3 m. We found the maximum theoretical conversion efficiency of a turbine is ~0.6 from the so-called Betz's law and the air density is about 1 kG/m^3.

a) What is the average wind-speed at this site? Here the average is a *time average* taken over a long operational period

b) What is the maximum power from this turbine?

c) The turbine design is such that it can only operate in winds with speeds that exceed 2 m/sec. What is the average power from this turbine? (again, here the average is a *time average* taken over a long operational period so that periods of both low speed and high speed wind are considered in the average power calculation)

d) Suppose the turbine costs $5,000, requires no maintenance and lasts 10 years (about 100,000 hours), and competing sources of electricity are available at a cost of $0.15/kW-hr of electrical energy. Does it make economic sense to purchase and install the wind turbine? You may neglect any interest costs (i.e. assume interest rates are zero).

a) $V_{avg} = \frac{10+1}{2} = \boxed{5.5 \text{ m/s}}$

b) $P_{max} = \eta\left[\frac{1}{2}A\rho V_{max}^3\right] = \frac{6}{10}\cdot\frac{1}{2}6\text{ m}^2\cdot 1\frac{\text{kg}}{\text{m}^3}\cdot 10^3\frac{\text{m}^3}{\text{s}^3}$

$A = (1.5)^2\pi \simeq 2.3 \simeq 6\text{m}^2$ $\quad = \frac{18}{10}\cdot 1000 \text{ W} = 1.8\text{ kW} \simeq \boxed{2\text{ kW}}$

c) $P_{ave} = \int_2^{10} \eta\frac{1}{2}\rho A V^3 \, P(V)\, dV$ where $P(V) = \begin{cases} 0.5 & V = 1, 10 \\ 0 & V \neq 1, 10 \end{cases}$

$\therefore P_{ave} = \eta\frac{1}{2}\rho A V^3\Big|_{V=10}(0.5) = \frac{6}{10}\cdot\frac{1}{2}\cdot\frac{1}{1}\cdot 6\cdot 1000\cdot\frac{1}{2} = \frac{9000}{10}$

$= 900\text{ W}$

$\boxed{\simeq 0.9\text{ kW}}$

d) $t = 10^5$ hr

$E_{total} = 0.9\text{ kW}\cdot 10^5\text{ hr} = 9\times 10^4\text{ kW-hr}$ over 10 years

$\text{cost} = \frac{\$5{,}000}{9\times 10^4\text{ kW-hr}} \approx \frac{\$5\times 10^3}{1\times 10^5\text{ kW-hr}} \simeq 5\times 10^{-2}\ \$/\text{kW-hr} \simeq \boxed{0.05\ \$/\text{kW-hr}}$

This is less than $0.15/kW-hr so it does make economic sense to purchase.