MAE 119 WINTER 2015
PROFESSOR G.R. TYNAN

QUIZ 1 CLOSED BOOK CLOSED NOTES

1. List three non-economic human quality-of-life metrics that are correlated with energy access which we discussed in lecture. (5 points each, 15 points total)

2. What are the top three primary energy sources in use globally today? (5 points each, 15 POINTS TOTAL)

3. Suppose a typical US resident consumes 4000 kW-hr of electrical energy per year. If this energy is produced by burning coal, how much carbon emission results from this energy consumption? Assume coal releases 30 MJ/kg, and that coal is 100% carbon, and that the coal is converted into electrical energy with an efficiency of 30%. An answer to one significant figure is sufficient. 15 points.

4. In lecture we saw data showing that the population growth rate, $r(t)$, is inversely related to per capita annual energy usage $E(t)$, i.e. $r(t) = r_0 \frac{E_0}{E(t)}$. Here at time $t=0$ the population is P_0 with a growth rate r_0 and per-capita energy usage E_0. For $t>0$, E is then increasing linearly in time, i.e. $E(t)=E_0\,(1+t/t_0)$.
    a. Write a differential equation giving the time derivative of the population in terms of r_0 and E_0 and t_0. 5 points
    b. Find an integral expression for $P(t)$. 5 points
    c. If the characteristic time, t_0, were to increase, how would the population at a fixed time, t, change? [HINT: You don't need to solve any equations to answer this, just use your result from part b above to see how this change affects the population growth] 5 points

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1. life expectancy
   Infant mortality
   Education level or literacy rate
   Population growth rate

2. coal, oil, natural gas

3. convert 4000 kWh to kJ: note $W = \frac{J}{s}$

$$4000\ kWh \cdot \frac{60\ min}{1h} \cdot \frac{60\ sec}{1\ min} = 14{,}400{,}000\ kJ$$

$$= 1.44 \times 10^{10} J \ (1 \times 10^{10} \text{ okay})$$

next find amount of coal needed to produce

$$\frac{14{,}400\ MJ}{30\ \frac{MJ}{Kg} \cdot 0.3} = \boxed{1600\ Kg} \quad \text{or} \quad \frac{1 \times 10^{4}\ MJ}{30\ \frac{MJ}{Kg} \cdot 0.3} = \frac{1 \times 10^{4}}{1 \times 10^{1}} = \boxed{1 \times 10^{3}\ Kg}$$

(0.3 ↑ at conversion efficiency 30%)

either answer is okay, only 1 sig-fig required

4. a) $\partial P = r(t) P(t)\, \partial t$

$$\hookrightarrow \frac{\partial P}{\partial t} = r_0 \frac{t_0}{t_0 \left(1 + \frac{t}{t_0}\right)} P(t)$$

$$\boxed{\frac{\partial P}{\partial t} = \left[\frac{r_0}{\left(1 + \frac{t}{t_0}\right)}\right] P(t)}$$

b) Seperate variables and integrate

$$\int_0^t \frac{1}{P(t)}\, dP = \int_0^t \frac{r_0}{(1+ t/t_0)}\, dt$$

$$\ln(P(t))\Big|_{t=0}^{t=t} = \int_0^t \frac{r_0}{(1+ t/t_0)}\, dt$$

$$\ln(P(t)) - \ln(P_0) = \int_0^t \frac{r_0}{(1+ t/t_0)}\, dt$$

$$\exp\left[\ \ln\left(\frac{P(t)}{P_0}\right) = \text{''}\ \right]$$

$$\boxed{P(t) = P_0 \exp\left[\int_0^t \frac{r_0}{(1+ t/t_0)}\, dt\right]}$$

c) As t_0 increases Energy growth rate increases at a slower rate, and subsequently population growth rate will decrease at slower rate. The result is that the population growth will level off more slowly, and reach a larger value.

The idea is that if we increase per capita energy use, we can actually reduce global energy demand in the long term by reducing population growth rate and stabilizing total population at a lower value.

by bringing rapid energy access to the existing pop.