MAE 119 Professor G.R. Tynan
Winter 2018
Quiz 3
Closed Book/Closed Notes. Calculators permitted.

In our simple carbon balance model, we assumed that the flux of C between the atmosphere and land & ocean was proportional to the deviation of carbon concentration from the equilibrium value. The atmospheric carbon balance model could then be written as

$$\frac{\partial}{\partial t}\delta M_C(t) = Q_C(t) - \frac{\delta M_C}{t_{net}}.$$

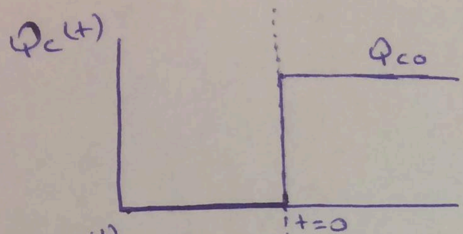
Where t_{net} denotes the effective timescale for C exchange with the Earth surface and oceans, $\delta M_C(t)$ denotes the deviation of the atmospheric carbon content away from the equilibrium value, and $Q_C(t)$ is the carbon injection rate from fossil fuel combustion. Suppose that the effective absorption time, t_{net}=100 year, and in the distant past when Q_C =0 the Earth's atmosphere contained CO_2 with a C mass of 600 Gigatonnes.

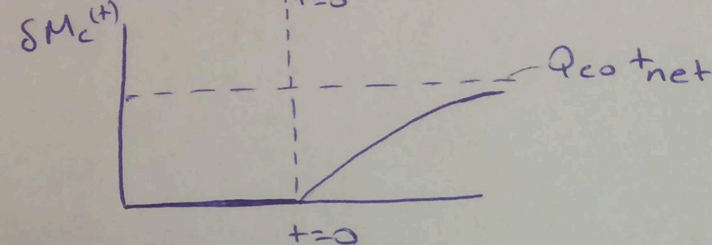
a) If $Q_C(t)$=0 for t<0 and $Q_C(t)$= Q_{C0}=constant for t>0, find the solution for $\delta M_C(t)$ for all t. 10 points.
b) For time $0<t<<t_{net}$ (i.e. shortly after the carbon injection rate started), how is $\delta M_C(t)$ related to Q_{C0} ? 10 points. Explain why this is the case. 5 points.
c) If C-containing molecules are the only greenhouse gas molecule of importance, and we allow the infra-red transmission coefficient of the atmosphere to decrease by a factor of 1/e for $t>>t_{net}$, what is the maximum allowable C source strength, Q_{C0} for this to occur?
(Note: you can take β_0=0.05 and an answer to only one significant figure will suffice) 10 points.

Quiz 3

a) $t < 0 \qquad Q_c(t) = 0$

$t > 0 \qquad Q_c(t) = Q_{co}$





$$t < 0 \quad Q_{co} = 0 \rightarrow \delta M_c = 0$$

$$\delta M_c(t) = Q_{co}\, t_{net} \left(1 - e^{-t/t_{net}}\right)$$

b) Shortly after the carbon injection rate started, δM_c doesn't change much initially for $t > 0$ but $t \ll 100$

$$\delta M_c \Big|_{t<0} \simeq \delta M_c \Big|_{\substack{t>0 \\ t \ll 100}} \simeq 0$$

$$\frac{\partial}{\partial t} \delta M_c(t) = Q_c(t)$$

$$\int_0^t \left(\frac{\partial}{\partial t} \delta M_c(t) = Q_{co} \right)$$

$$\delta M_c(t) = Q_{co}\, t$$

Deviation of the atmospheric carbon content away from the equilibrium value is linear with time t. for time $0 < t \ll t_{net}$

c) $t >> t_{net}$

$$\beta(t) = \beta_o \exp\left(-\sigma d \, \delta n(t)\right)$$

$$\beta_o\left(1-\frac{1}{e}\right) = \beta_o \exp\left(-\frac{\sigma d}{m_c V} \delta M_c(t)\right)$$

$$\beta_o\left(1-\frac{1}{e}\right) = \beta_o \exp\left(\frac{\ln \beta_o}{M_o} \delta M_c(t)\right)$$

$M_o = 600$ Gigatonnes

δM_c saturates at $\varphi_{co} \, t_{net}$

$\beta_o = 0{,}05$

$$1-\frac{1}{e} = \exp\left(\frac{-3}{600} \varphi_{co} \, 100\right)$$

$\varphi_{co} = 0{,}917$ Gigatonnes / year