MAE 119 Winter 2018 Professor G.R. Tynan
Quiz 5: Wind Energy

Closed Book/Closed Notes. Calculators Permitted

1. A wind turbine with blades that sweep out an area of 1000 m^2 extracts power from the wind. The upstream wind speed is measured to be 10 m/sec, and the wind speed downstream of the turbine has a value of 5 m/sec. If this turbine operates with a power factor C_p=0.4, estimate the power that the turbine produces. One significant figure will suffice. 10 points.

2. The wind speed at site A has an equal probability of lying between 0 < V < 10 m/sec; at this site the wind speed never exceeds 10 m/sec. The wind at site B always blows at 5 m/sec. You have a turbine available that can safely operate for wind speeds anywhere between 0 m/sec and 15 m/sec. At which site should you play the turbine to maximize the energy produced over a long period of time? 10 points.

3. What is the capacity factor of a wind turbine placed at site A in problem 2 above ? 10 points.

Quiz 5

1) $P = \frac{1}{2} \rho A_t V_t (V^2 - V_w^2)$

$V_t = \frac{1}{2}(V + V_w) = \frac{1}{2}(10+5) = \frac{15}{2}$ (m/s)

$P_{max} = \frac{1}{2} \cdot 1000 \cdot \frac{15}{2} (10^2 - 5^2) = 281250$ (W)

$P = P_{max} C_p = 112500$ (W)

2) Site A:

$$P_{ave} = \frac{\int_0^{10} \frac{1}{2} \rho A V^3 f(V)\, dV}{\int_0^{10} f(V)\, dV} = \frac{\int_0^{10} \frac{1}{2} \cdot 1000\, V^3 \cdot 0.1\, dV}{1}$$

$$P_{ave} = \frac{1}{2} \cdot 1000 \cdot 0.1 \cdot \frac{10^4}{4} = 125000 \text{ (W)}$$

Site B:

Note: Assuming $A_t = 1000\ m^2$ for this question.

$V = 5$ m/s

$$P = \frac{16}{27} \cdot \frac{1}{2} \rho A V^3 = \frac{16}{27} \cdot \frac{1}{2} \cdot 1000 \cdot 5^3 = 37037 \text{ (W)}$$

Site A should be used to maximize the energy produced over a long period of time.

3) $P_{max} = \frac{16}{27} \cdot \frac{1}{2} \rho A V_{max}^3 = \frac{16}{27} \cdot \frac{1}{2} \cdot 1000 \cdot 10^3 = 296296.3$ (W)

$P_{ave} = C_F P_{max}$

$125000 = C_F \cdot 296296.3$

$C_F = 0.42$