UC SAN DIEGO MAE 119 FINAL EXAM
PROFESSOR G.R. TYNAN

NO ELECTRONIC DEVICES PERMITTED. STUDENTS MAY REFER TO THEIR EQUATION SHEETS BROUGHT TO THE EXAM.

PART I: MULTIPLE CHOICE. ALL PROBLEMS 2 points each.

1. Which of the following are primary energy sources used today by human beings?
   a. Coal, Petroleum, Electricity, Gasoline
   b. Petroleum, Fissile Uranium, Natural Gas
   c. Natural Gas, Diesel, Geothermal Energy, Biofuels
   d. None of the above.
2. Which fuel has the lowestcarbon per unit energy released?
   a. Coal
   b. Natural Gas
   c. Petroleum
   d. Wood
3. A mass of 100 kG is accelerated by a force of 100 N for 10 seconds. How much work is done on the mass?
   a. 100 J
   b. 50 J
   c. 500 J
   d. none of the above.
4. The first law of thermodynamics states that
   a. Entropy must always increase
   b. Energy is always conserved
   c. Using energy up allows us to do work on a system
   d. None of the above.
5. Which of the following are secondary energy sources in use today?
   a. Nuclear fission, hydropower and wind power
   b. Petroleum, Coal and Natural Gas
   c. Gasoline, Diesel and Electricity
   d. Electricity, Refined Fuels and Wood.
6. What is the relative ordering (from largest to smallest contribution to human energy demand) of primary energy sources?
   a. Fossil fuels, nuclear fission, and wind energy
   b. Solar thermal, solar photovoltaic and wind energy
   c. Gasoline, Diesel and biofuels
   d. Electricity, Heat and Liquid Fuels
7. We learned in our carbon balance discussion that CO2 emitted by combustion processes stays in the atmosphere for a period of time before being absorbed by the Earth's biosphere and land/ocean masses. The typical timescale for this reabsorption to occur is about:
   a. 10 years

    b. 50 years
    c. 100-200 years
    d. It stays in the atmosphere forever.
8. What is the order-of-magnitude of carbon-free power needed to meet future energy demands while avoiding significant climate change effects? Here we are lumping all types of power demand into one total rate of energy consumption.
    a. 1-2 TW
    b. a few GW
    c. 20 TW
    d. 100 TW
9. The capacity factor of an electricity generation technology is defined as:
    a. The maximum amount of power than the technology can produce
    b. The maximum feasible contribution that the technology can contribute to human energy demand.
    c. The ratio of the maximum power produced to the minimum possible power produced by the technology.
    d. The ratio of the average power produced divided by the peak power produced by the technology.
10. How did we measure the means by which access to adequate energy resources improve human quality of life?
    a. It frees up people to pursue other interests besides growing enough food to survive.
    b. It impacts multiple social measures including literacy rates and childhood mortality.
    c. It is correlated with increased human lifespan, economic development and average number of years in school.
    d. All the above.

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PART II:

- **ALL PROBLEMS ARE EQUALLY WEIGHTED AT 20 POINTS EACH.**
- **SOLVE ANY FIVE (5) PROBLEMS.**
- **IF A NUMERICAL ANSWER IS REQUIRED, ONE SIGNIFICANT FIGURE WILL SUFFICE.**

1. *Tidal power:* An area of uniform water depth is being considered for development as a tidal power basin. The water height undergoes a sinusoidal oscillation in height. The period of oscillation is 12 hours, and the surface height variation is 2 m from peak to trough. A 10 km^2 area is planned to be developed into a tidal basin.
   a. How much energy can be stored in this basin?
   b. If this energy is released over a 2 hour period when the surrounding water is at low tide, estimate how much power could be generated during this period of time? Assume that the conversion apparatus has 100% efficiency.

2. *Deep Geothermal Power:* A heat mine is to be developed at a site. The mine will access rock that can be fractured in a region with a geothermal temperature gradient of 20 deg K/km of depth. The mine will exploit a region 10 km x 10 km on the surface, and will have a vertical extent of 100 m at an average depth of 5 km. The rock has a density of 10 Tonnes/m^3 and a thermal capacity of 2000 J/kG-deg C and the ambient surface temperature is 300 K.
   a. How much thermal energy can be extracted from the 10 km x 10 km + 100 m volume of rock?
   b. If the heat is extracted uniformly throughout this volume of rock at a rate P_{out}, write a zero-dimensional time-dependent energy balance equation for the volume, which shows how the rock temperature evolves in time. (Hint: You should get a first-order time-dependent ordinary differential equation).
   c. Solve this equation to get a solution for the rock temperature evolution vs. time. Hint: Look for exponential-like solutions.
   d. Find an expression for the time needed for the rock temperature to drop by a factor of 1/e? If P_{out}=100 MW, estimate this time scale.

3. *Solar PV:* A p-n junction based solar cell has an open-circuit voltage of 0.6 V and a short-circuit current of 20 mA/cm^2 when exposed to a solar insolation of 1000 W/m^2, and has a form factor FF = 0.7.

a. Draw the current-voltage characteristic of this device, making sure to identify the V_{OC} and I_{SC} locations, and the point where the maximum power is generated.
b. What is the efficiency of this cell?
c. Suppose you can improve the manufacturing process of the cell so that the carrier lifetime is increased by 4x. What will happen to the V_{OC} and I_{SC} of the cell? Draw how the current-voltage characteristic will change.
d. If the solar insolation of 1000 W/m^2 is constant for 8 hours/day and then is zero for 16 hours/day what is the capacity factor of this cell? How much electrical energy will it produce in a day?

4. *Climate change essentials: Earth's heat balance*
   a. A perfectly absorbing surface is exposed to a visible light solar insolation. If there is no atmosphere to absorb light, what is the infra-red emissivity of the surface in steady-state?
   b. If the surface emits as a perfect blackbody, what is it's equilibrium temperature?
   c. Suppose that an atmosphere is now introduced just above this surface. The atmosphere is perfectly transparent to visible light, but absorbs 50% of infra-red radiation that passes through it. This warms the atmosphere which then re-radiates half of this radiation to space, and half back to the surface. What are the equilibrium surface and atmospheric temperatures?
   d. How do these compare to the temperature in part (b)?

5. *Climate change essentials: Carbon balance modeling:* An atmosphere has an infra-red absorbing species injected into it at a constant rate Q starting at t=0. Prior to t=0 there was no such species in the atmosphere. This species persists in the atmosphere for a timescale τ before it is reabsorbed by the surface lying beneath the atmosphere.
   a. Draw a simple 0-d control volume model showing the atmosphere containing a mass M of the species in question, the source rate, and the reabsorption rate of the species, Γ, by the planetary surface.
   b. Use this model to write a time-dependent 0-d mass balance equation for M(t) in the atmosphere.
   c. If the reabsorption rate $\Gamma = M(t)/\tau$, find M(t) for t>0.
   d. If the atmosphere has a height d, and the injected species with an IR absorption cross-section σ is uniformly mixed in the atmosphere, find an expression for the IR transmission coefficient vs time of the atmosphere.

6. *Wind Power:* A region has a wind-speed probability distribution function $f(v)=0.1$ for $0<V<10$ m/sec and $f=0$ for $V>10$ m/sec.
    a. What is the probability that the wind speed will satisfy $5<V<6$ m/sec at any given time?
    b. Write an integral expression for the average wind speed, and solve it to find the average wind speed. If the wind was to always blow at this speed, what would be the power per unit area incident on a wind turbine?
    c. Write an integral expression for the average power density available at the face of a wind turbine located in this region. Solve it to find the average power density. Compare this average power to what you computed in part (b). Why are the two different?
    d. If you have a wind turbine available with a cut-in speed of 3 m/sec and a cut-out speed of 8 m/sec, how often will the wind-turbine operate? How often will it be off-line?

7. *Solar Thermal:* A solar power tower design concept has a centrally located point-like target illuminated by an array of heliostat mirrors that can be oriented to reflect sunlight onto the target. The ratio of the mirror collecting area to the target area is a factor of 1000.
    a. If the direct normal incidence (DNI) solar irradiation is 1000 W/m^2, what is the incident heat flux to the target?
    b. If the working fluid of the power plant removes heat from the target at a rate of 500 kW/m^2, what will be the equilibrium temperature of the target? [Hint: write a power balance for a unit surface area of the target and then recall that the emitted heat flux from a radiating body goes like σT^4 where σ denotes the Stefan-Boltzmann constant which has a value of ~6x10-8 W/m^2-K^4]
    c. If the working fluid has a temperature that is half of the target temperature, estimate the thermal conversion efficiency of an ideal heat engine deployed in this system.
    d. Suppose a cloud layer moves over that has a thickness of 1km. The cloud is composed of aerosol particles with a cross-sectional area of 10^{-9} m^2. These particles have a density of 10^6 particles/m^3. What is the DNI now? If the plant were to keep operating, by how much will the power plant power output decline (you may neglect any change in the thermal conversion efficiency).

PART I.

1) B
2) B
3) D
4) B
5) C
6) A
7) C
8) C
9) D

PART II.

1. Tidal Power

a) The potential of a fluid element δm has a potential energy of

$$dU = \delta m \cdot g \cdot y.$$

at height of ~~h~~ y.

Pluging $\delta m = \rho \cdot A \cdot g \cdot \delta y.$ in, we have.

$$dU = \rho g A \, y \, dy \Rightarrow U = \int_0^{h=2.} \rho g A y \, dy.$$

$$\therefore \; U = \frac{1}{2}\rho g A h^2 . = \frac{1}{2} \times 10^3 \times 10 \times (10 \times 10^6) \times 4 = 2 \times 10^{11} \text{ (J)}$$

b) Power released is $P = \dfrac{U}{T} = \dfrac{2 \times 10^{11}}{2 \times 3600} \approx 2.5 \times 10^7 \text{ (W.)}$

2. Deep Geothermal Power.

a). $\nabla T = 20\ K/km. \quad H = 5\ km.$

$\Delta T = H \cdot \nabla T = 100\ (K)$.

$E = c \cdot m \cdot \Delta T = 2\times10^3 \times 10\times10^3 \times (10\times10^3)^2 \times 100 \times 100$

$= 2\times10^{18}\ (J)$.

(b). Time-dependent energy conservation equation is

$$\frac{\partial}{\partial t}(\rho C_p T) + \nabla\cdot Q = P_{in} - P_{out}.$$

where Q is the heat flux.

Integrating over the whole volume V, we have.

$$\rho C_p \int_V \frac{\partial T}{\partial t} dV + \int_V \nabla\cdot Q\, dV = \int_V (P_{in} - P_{out})\, dV.$$

$$\Rightarrow \rho C_p \int_V \frac{\partial T}{\partial t} dV + \oint_{\partial S} Q\cdot dA = \int_V (\cancel{P_{in}}^{\,0} - P_{out})\, dV.$$

$\oint Q\cdot dV \sim Q_r \cdot A_r$ is usually small compared to other terms

Denotes. $\bar{T} = \frac{1}{V}\int T\, dV$, $\cancel{\bar{P}_{in} = \frac{1}{V}\int P_{in}\, dV}^{\,0}$, $P_{out}^{tot} = \int P_{out}\, dV$

We have. $\boxed{\rho V C_p \frac{\partial \bar{T}}{\partial t} = -P_{out}^{tot.}}$

2 cont'd.

(c). Letting $P_{out}^{tot} = \frac{C_p m \cdot \Delta \bar{T}}{\tau}$ where $m = \rho V$.

The equation becomes

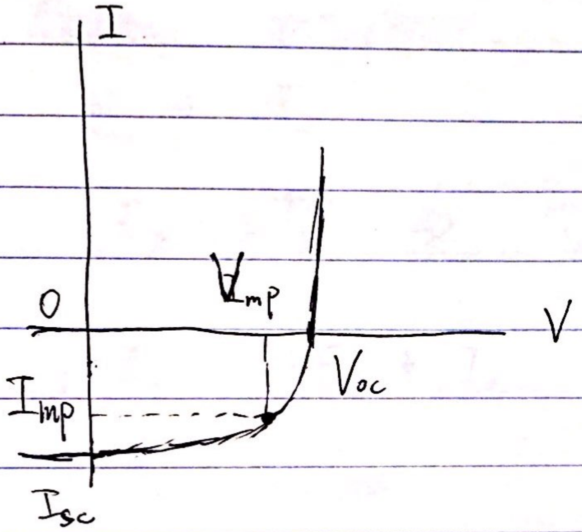
$$\frac{\partial \bar{T}}{\partial t} = -\frac{\Delta T}{\tau} \Rightarrow \bar{T} \propto e^{-t/\tau}.$$

(d). $\tau = \frac{C_p \cdot \rho V \cdot \Delta \bar{T}}{P_{out}^{tot}} = \frac{E}{P_{out}^{tot}} = 2 \times 10^{10} (s) \approx 600 (year).$

(5)

3. Solar PV.

a)

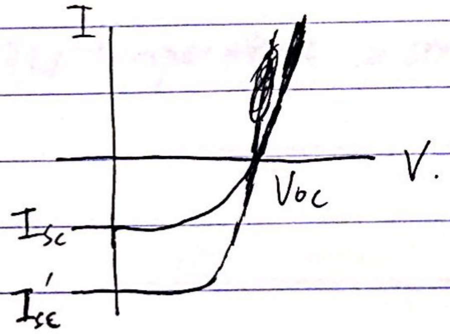


$$FF = \frac{I_{mp} V_{mp}}{I_{sc} V_{oc}}$$

b) $$\eta = \frac{FF \cdot I_{sc} \cdot V_{oc}}{I_0} = \frac{0.7 \times 200 \times 0.6}{1000} = 8.4\%$$

c) V_{oc} keeps ~~un~~changed.

$I_{sc} \propto \sqrt{I_{oc}}$, increases by twice.



d) $$C = \frac{8}{24} = \frac{1}{3}.$$

$$P = FF \cdot I_{sc} \cdot V_{oc} = 84\ W/m^2$$

$$E = P \cdot \Delta t = 0.67\ kWh/m^2.$$ per day.

4) Heat balance.

a). Output = Input power.

$E_s = I_0$.

b). $E_s = \sigma T_s^4 \Rightarrow T_s = \left(\frac{I_0}{\sigma}\right)^{1/4}$.

c). $\begin{cases} E_s = 0.5E_a + I_0 \\ E_a = 0.5E_s \end{cases}$

$\Rightarrow$ ~~$E_s = \frac{1}{4}E$~~ $\begin{cases} E_s = \frac{4}{3}I_0 \\ E_a = \frac{2}{3}I_0 \end{cases}$

$\Rightarrow \begin{cases} T_s = \left(\frac{4}{3}\frac{I_0}{\sigma}\right)^{1/4} \\ T_a = \left(\frac{2}{3}\frac{I_0}{\sigma}\right)^{1/4} \end{cases}$.

d). The temperature increases. when atmosphere is introduced.

5. Carbon balance.

a) $\cancel{\frac{\partial}{\partial t} M = Q - \Gamma}$.

$$
\begin{tikzcd}
\boxed{\text{Q, Injection rate of C}} \arrow[r] & \boxed{M_c\text{, Atmosphere C mass}} \arrow[d, "\Gamma"] \\
& \boxed{M_s\text{, Surface C Mass}}
\end{tikzcd}
$$

b) $$\frac{\partial M_c(t)}{\partial t} = Q - \Gamma.$$

c) $$\frac{\partial}{\partial t} M_c(t) = Q - \frac{M_c(t)}{\tau}.$$

$$\Rightarrow M_c(t) = M_c(0)\, e^{-t/\tau} + Q\cdot\tau.$$

d) $$\frac{dI(x)}{I(x)} = -\sigma n \cdot dx.$$ (from x to $x+dx$, relative change of intensity is $\frac{dI(x)}{I(x)}$.)

$$\Rightarrow \frac{I(d)}{I(0)} = e^{-\sigma n d}.$$

$$\beta \equiv \frac{I(d)}{I(0)} = e^{-\sigma n d}, \qquad n = \frac{M_c}{V\cdot m_c}$$ m_c = carbon atom mass

$$\cancel{\therefore n(t) = \frac{1}{V} = \frac{\rho}{M(t)}}$$

$$\cancel{\therefore \boxed{\beta(t) = \exp\left(-\frac{\sigma \rho d}{M(t)}\right)}}$$

$$\frac{\ln \beta(0)}{M_c(0)} = \frac{-\sigma d}{V\cdot m_c}$$

$$\Rightarrow \beta(t) = \beta_0 \exp\left[\frac{\ln\beta(0)}{M_c(0)}\left(M_c(t) - M_c(0)\right)\right]$$

(8)

6. Wind Power.

a) $$\text{Prob} = \frac{\int_5^6 f(v)\,dv}{\int_0^\infty f(v)\,dv} = \frac{0.1V\big|_5^6}{0.1\cdot V\big|_0^{10}} = \frac{1}{10}.$$

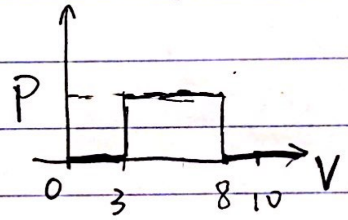
b). $$V_{ave} = \frac{\int_0^\infty v f(v)\,dv}{\int_0^\infty f(v)\,dv} = \frac{1}{2}\times\frac{1}{10}V^2\Big|_0^{10} = 5\ m/s.$$

$$\frac{P}{A}(V=V_{ave}) = \frac{1}{2}\eta\rho V_{ave}^3 = \frac{125}{2}\eta\rho.$$

c). $$\frac{P_{ave}}{A} = \frac{\frac{1}{2}\eta\rho\int_0^\infty V^3\cdot f(v)\,dv}{\int_0^\infty f(v)\,dv} = \frac{1}{2}\eta\rho\cdot\frac{1}{40}V^4\Big|_0^{10} = 125\eta\rho.$$

$P_{ave} > P(V=V_{ave})$. since. $P \propto V^3$, large speed weights more.

d) Power of turbine.



$$\text{Prob} = \frac{\int_3^8 f(v)\,dv}{\int_0^\infty f(v)\,dv} = \frac{1}{2}.$$

Works $\frac{1}{2}$ time. and $\frac{1}{2}$ time off-line.

7. Solar Thermal.

a). $P_{in} = C \cdot I_0 = 1000 \times 1000 = 10^6 \, (W/m^2.)$

b). Power balance eqn. is

$$C \cdot I_0 = \sigma T^4 + P_{out}.$$

$$\Rightarrow T = \left(\frac{C I_0 - P_{out}}{\sigma}\right)^{1/4} = \left(\frac{10^6 - 5\times10^5}{6\times10^{-8}}\right)^{1/4} \approx 1700K$$

c). $\eta = 1 - \frac{T_c}{T_H} = 1 - 0.5 = \underline{0.5}.$

d). $$\frac{I(d)}{I(0)} = e^{-\sigma n d} = \exp[10^{-9} \times 10^6 \times 10^3] = e^{-1}$$

$$\approx 0.36.$$

The decline is about $1 - 0.36 = 64\%$.