MAE 119 Professor G.R. Tynan
Winter 2013
Quiz 3
Open Book/Open Notes

In our simple carbon balance model, we assumed that the flux of C between the atmosphere and land & ocean was proportional to the deviation of carbon concentration from the equilibrium value. The atmospheric carbon balance model could then be written as

$$\frac{\partial}{\partial t}\delta M_C(t) = Q_C(t) - \frac{\delta M_C}{t_{net}}.$$

Where $t_{net} = \dfrac{t_1 t_2}{t_1 + t_2}$ denotes the effective timescale for C exchange with the Earth surface and oceans, $\delta M_C(t)$ denotes the deviation of the atmospheric carbon content away from the equilibrium value, and $Q_C(t)$ is the carbon injection rate from fossil fuel combustion.

Suppose the carbon source, $Q_C(t<0)=8$ GigaTonnes/year,is constant for time $t<0$, and then at $t=0$ the carbon source injection rate is doubled so that for $t>0$ $Q_C(t>0)=16$ GigaTonnes/year. The effective absorption timescale is given as $t_{net} = 100\,years$.

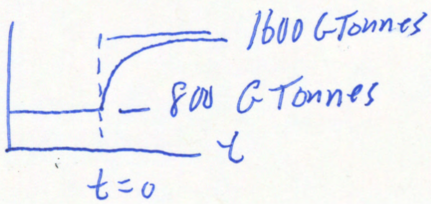
a) For very early times (i.e. $t << 0$) what is δM_C? 5 POINTS
b) For very late times (i.e. $t >> 0$) what is δM_C? 5 POINTS
c) Sketch the time evolution of δM_C. How long will it take for δM_C to get within about 70% of its final value after the change in injection rate at t=0? One significant figure will suffice. 5 POINTS
d) For very late times (i.e. $t >> 0$) what will be the value of the IR transmission coefficient *relative* to the value it had at very early times (i.e. $t<<0$)? 5 POINTS

EXTRA CREDIT (10 POINTS):

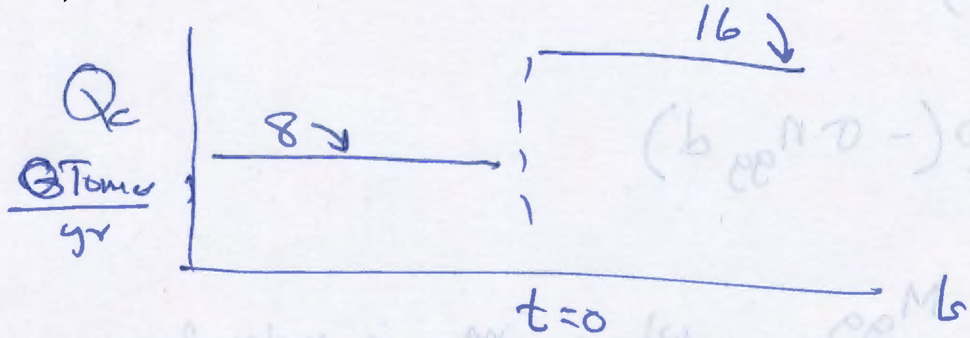
Find the time evolution of $\delta M_C(t)$. *Hint:* You don't have to solve the model ODE. Instead, note that this model is *linear* and so solutions and C sources can be superimposed.

a) $\delta M_c = Q_{c_0} t_{eff}$ for this case. Thus $\delta M_c\big|_{t<0} = 8 \cdot 100 = 800$ GTonnes

b) $\delta M_c = Q_{c,0} t_{eff}$ " $\delta M_c\big|_{t>>0} = 1600$ GTonnes

c) 

It will take about $1 \times t_{eff} = 100$ years for the δM_c value to get to $\simeq 70\%$ of its final value



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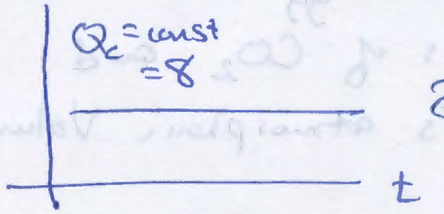

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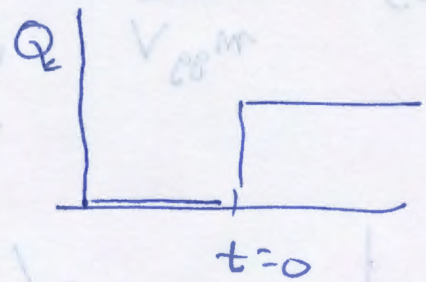

Solution:

$$\delta M_c = Q_{c_0} t_{net}$$

Solution:

$$\delta M_c = Q_{c_0} t_{net} \left(1 - e^{-t/t_{net}}\right) \quad t>0$$

$$\delta M_c = 0 \quad t<0$$

$\therefore$ total solution

$$\delta M_c = Q_{c_0} t_{net} + Q_{c_0} t_{net} \left(1 - e^{-t/t_{net}}\right) \quad t>0$$

$$\boxed{\delta M_c = Q_{c_0} t_{net} \left(2 - e^{-t/t_{net}}\right) \quad t>0}$$

$$\boxed{\delta M_c = Q_{c_0} t_{net} ; \quad t<0}$$

Quiz 3 (cont'd)

(d) $\beta_{IR} = \exp(-\sigma n_{gg} d)$

$n_{gg} = \dfrac{M_{gg}}{m_{gg} V}$ where M_{gg} is molecular mass of CO_2 and V is atmospheric Volume

thus

$$\frac{\beta_{IR}\big|_{t\gg 0}}{\beta_{IR}\big|_{t\ll 0}} = \frac{\exp\left(-\left(\frac{\sigma d}{m_{gg}V}\right) M_{gg}\big|_{t\gg 0}\right)}{\exp\left(-\left(\frac{\sigma d}{m_{gg}V}\right) M_{gg}\big|_{t\ll 0}\right)}$$

$$= \exp\left(-\frac{\sigma d}{m_{gg}V}\left(M_{gg}\big|_{t\gg 0} - M_{gg}\big|_{t\ll 0}\right)\right)$$

but $M_{gg}\big|_{t\gg 0} = 2 M_{gg}\big|_{t\ll 0}$

$$\therefore \frac{\beta_{IR}\big|_{t\gg 0}}{\beta_{IR}\big|_{t\ll 0}} = \exp\left(-\left(\frac{\sigma d}{m_{gg}V}\right) M_{gg}\big|_{t\ll 0}\right) = \beta_{IR}\big|_{t\ll 0}$$

$$\therefore \boxed{\beta_{IR}\big|_{t\gg 0} = \left(\beta_{IR}\big|_{t\ll 0}\right)^2}$$