MAE 119   Professor G.R. Tynan
Winter 2013 Quiz 4
Open Book/Open Notes

In a solar thermal power system we saw that the temperature of the working fluid obeys the equation

$$\rho C_p V \frac{\partial T}{\partial t} = -\frac{P_{out}}{\eta_{th}}$$

when there is no power input to the system, e.g. when the sun is not shining or the system is otherwise not collecting sunlight. In this equation ρ denotes the fluid mass density, C_p denotes the fluid specific heat at constant pressure, V denotes the volume of the storage tank incorporated into the solar thermal system, $\frac{\partial T}{\partial t}$ denotes the time derivative of the fluid temperature, η_{th} is the thermal conversion efficiency, and P_{out} is the *power* output of the system during such a state.

a) If there ***is*** power input into the system from the sun, which we denote as P_{in} what would be the new energy balance equation? In order words, how would you modify the equation above to account for the power input? (5 points)

We just need to add the new power input term to the RHS of the equation:

$$A = \frac{P_{out}}{I\eta_{th}} = \frac{10^8 Watts}{300W/m^2 * 0.3} \sim 1.1x10^6 m^2$$

b) If this input power from the sun is constant in time, and the system had zero output power, what would the functional form of T(t) be? (5 points)

If P_{in} is constant, and there is no power output, then this equation becomes

$$\rho C_p V \frac{\partial T}{\partial t} = P_{in}$$

for convenience we can define a constant, k, as

$$k = \frac{P_{in}}{\rho C_p V}$$

and the energy balance equation then becomes simply

$\frac{\partial T}{\partial t} = k$ where k is a constant. Thus we can integrate to find T(t):

$T(t) = T\big|_{t=0} + kt$.

This corresponds to a linearly increasing temperature vs. time.

c) If the solar intensity is 300 W/m^2 and the thermal conversion efficiency is 30%, what is the area of the collecting mirrors required for a 100 MW output power? You may assume that the plant would be operating in steady-state. (5 points)

For this scenario, we start with the equation from part (a):

$$\rho C_p V \frac{\partial T}{\partial t} = P_{in} - \frac{P_{out}}{\eta_{th}}$$

we are told that the plant is operating in steady state. Thus the LHS vanishes and we can then write

$$P_{in} = \frac{P_{out}}{\eta_{th}}$$

With the input power given in terms of the mirror area A and the solar incident radiation intensity I we then have

$$IA = \frac{P_{out}}{\eta_{th}}$$

which we can solve for the mirror collecting area:

$$A = \frac{P_{out}}{I\eta_{th}} = \frac{10^8 Watts}{300W/m^2 * 0.3} \sim 1.1x10^6 m^2$$

Realistically there needs to be some open land area between collecting mirrors, central tower and power conversion equipment, etc… So a reasonable estimate for the land area is in the range of 1-2 x 10^6 m^2 which is about 1-2 km^2

Extra credit (5 points):

Suppose we want to add additional mirrors to the plant described in part (c) so that we can store the excess heat produced in the working fluid so that the plant can produce 100 MW output power at night. If day and night have equal length, and we assume constant sunlight intensity all through the day (certainly an idealization), what collecting area is needed now?

The energy collected and stored for delivery at night is equal to the energy collected and delivered during the daytime for this scenario. Thus the plant area is simply doubled, i.e. the total area estimate would lie in the range of 2-4 km^2.