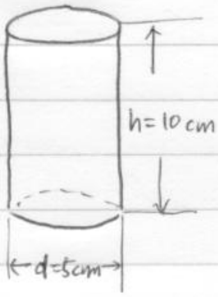
Solution 125B

HW2

Pb 1



$w = 350g$

$\rho_s = 2.65 g/cm^3$

$w(\text{porosity}) = \dfrac{V_{voids}}{V_{total}}$ $\qquad V_{total} = V_{solid} + V_{void}$

$$V_{total} = \pi \left(\frac{d}{2}\right)^2 h = \pi \times (2.5\ cm)^2 \times 10\ cm = 196.3495\ cm^3 \approx 196.35\ cm^3$$

$$V_{solid} = \frac{350g}{2.65 g/cm^3} = 132.0755\ cm^3 \approx 132.08\ cm^3$$

$$V_{void} = \cancel{64.27}\ 64.27 cm^3$$

$$w = \frac{64.27 cm^3}{196.35 cm^3} = 0.327324 \approx \underline{\underline{0.327}}$$

Pb2.

$$K = \frac{aL}{At} \ln\left(\frac{H_1}{H_0}\right)$$

a: cross-section of tube, L: length, $H = h_2 - h_1$

A: Area of cross-section, $Q(t)$: Mass flux, $Q(t) = a v(t)$

$v(t) = -\dfrac{dH}{dt}$

Darcy's law : $Q(t) = -K \dfrac{AH}{L}$

$$Q(t) = a v(t) = -a \frac{dH}{dt}$$

$$\rightarrow \quad K \frac{AH}{L} = a \frac{dH}{dt}$$

$$K \frac{A}{aL} dt = \frac{1}{H} dH$$

$$\int_0^t K \frac{A}{aL} dt = \int_{H_0}^{H_1} \frac{1}{H} dH$$

$$K \frac{A}{aL} t = \ln H_1 - \ln H_0$$

$$K = \frac{aL}{At} \ln\left(\frac{H_1}{H_0}\right)$$

Pb 3. (Eq 2.58) $u_1 = \frac{\rho g}{2\mu} J x_2^2 + a_1 x_2 + a_2$, $J = \frac{\partial h}{\partial x_1}$

(Eq 2.60) $u_1(x_2) = -\frac{\rho g}{\mu}\frac{1}{2}\left(\frac{b^2}{4} - x_2^2\right)\frac{dh}{dx_1}$

<u>B.C</u> $u_1\left(-\frac{b}{2}\right) = u_1\left(\frac{b}{2}\right) = 0$

$$u_1\left(\frac{-b}{2}\right) = \cancel{\frac{\rho g}{2\mu} J\left(-\frac{b}{2}\right)^2} - a_1\frac{b}{2} + \cancel{a_2} = \cancel{\frac{\rho g}{2\mu} J\left(\frac{b}{2}\right)^2} + a_1\frac{b}{2} + \cancel{a_2} = u_1\left(\frac{b}{2}\right)$$

$\rightarrow \boxed{a_1 = 0}$

$$u_1\left(\frac{b}{2}\right) = \frac{\rho g}{2\mu} J\left(\frac{b}{2}\right)^2 + \cancelto{0}{a_1\left(\frac{b}{2}\right)} + a_2 = 0$$

$\rightarrow \boxed{a_2 = -\frac{\rho g}{8\mu} J b^2}$

$$u_1(x_2) = \frac{\rho g}{2\mu} J x_2^2 + \left(-\frac{\rho g}{8\mu} b^2\right) J$$

$$= -\frac{\rho g}{\mu}\frac{1}{2}\left(\frac{b^2}{4} - x_2^2\right) J = \boxed{-\frac{\rho g}{\mu}\frac{1}{2}\left(\frac{b^2}{4} - x_2^2\right)\frac{dh}{dx_1}}$$

Pb 4. $u_1(x_2) = -\frac{\rho g}{\mu}\frac{1}{2}\left(\frac{b^2}{4} - x_2^2\right)\frac{dh}{dx_1} \longrightarrow q_1 = \frac{1}{b}\int_{-\frac{b}{2}}^{\frac{b}{2}} u_1(x_2)\,dx_2 \Rightarrow -\frac{\rho g}{\mu}\frac{b^2}{12}\nabla h = \underset{\sim}{q}$

$$q_1 = \frac{1}{b}\int_{-\frac{b}{2}}^{\frac{b}{2}}\left[-\frac{\rho g}{\mu}\frac{1}{2}\left(\frac{b^2}{4} - x_2^2\right)\frac{dh}{dx_1}\right]dx_2$$

$$= -\frac{1}{2b}\left(\frac{\rho g}{\mu}\right)\frac{dh}{dx_1}\left[\int_{-\frac{b}{2}}^{\frac{b}{2}}\frac{b^2}{4}\,dx_2 - \int_{-\frac{b}{2}}^{\frac{b}{2}} x_2^2\,dx_2\right]$$

$$= -\frac{1}{2b}\left(\frac{\rho g}{\mu}\right)\frac{dh}{dx_1}\left[\frac{b^2}{4}\cdot b - \left[\frac{x_2^3}{3}\right]_{-\frac{b}{2}}^{\frac{b}{2}}\right] = -\frac{1}{2\cancel{b}}\left(\frac{\rho g}{\mu}\right)\frac{dh}{dx_1}\left[\frac{b^{\cancel{3}\,2}}{4} - \left(\frac{b^{\cancel{3}\,2}}{\cancel{24}_{12}}\times\cancel{2}\right)\right]$$

$$= -\frac{1}{\cancel{2}}\left(\frac{\rho g}{\mu}\right)\frac{dh}{dx_1}\left(\frac{\cancel{2}}{12}b^2\right) = -\frac{\rho g}{\mu}\frac{b^2}{12}\frac{dh}{dx_1}$$

$q_2 = -\frac{\rho g}{\mu}\frac{b^2}{12}\frac{dh}{dx_2}$ $\quad q_3 = -\frac{\rho g}{\mu}\frac{b^2}{12}\frac{dh}{dx_3}$

$$\Rightarrow \underset{\sim}{q} = (q_1, q_2, q_3) = -\frac{\rho g}{\mu}\frac{b^2}{12}\left(\frac{dh}{dx_1}, \frac{dh}{dx_2}, \frac{dh}{dx_3}\right) = \boxed{-\frac{\rho g}{\mu}\frac{b^2}{12}\nabla h}$$