MAE123, Assignment 4

Problem 1 (25 points)

The data below were obtained by gravimetric sampling just prior to and two days following an irrigation.

Sampling time	Sampling number	Bulk density (gm/cm^3)	Weight of wet sample + container (gm)	Weight of dried sample + container (gm)	Weight of container (gm)
Before irrigation	1	1.2	160	150	50
	2	1.5	145	130	50
After irrigation	3	1.2	230	200	50
	4	1.5	206	170	50

From these data, calculate the mass wetness and volumetric water content values of each sample before and after the irrigation.

Solution: The solution follows directly from the definitions of mass wetness and volumetric water content.

Problem 2 (40 points)

The data below were obtained with tension plate device from two soils of unknown texture.

Suction (cm)	Water content	
	Soil A	Soil B
0	44	52
10	44	52
20	43.9	52
50	38	51
100	22.5	48
300	12.5	32
1000	7	20
10,000	5.2	13.5
20,000	5.1	13
100,000	4.9	12.8

Plot the two soil-moisture characteristic curves on a semi-log scale (logarithm of suction versus water content). In addition, answer the following questions:

1. Which soil has a higher porosity?

2. What is the bulk density of the two samples? (Assume that the density of solids comprising both samples is $\rho_s = 2.65 gm/cm^3$.)

3. What is an estimated water content in the two samples at suction $15cm$?

Solution.

1. At full saturation, suction $\psi = 0$ and water content equals porosity, $\theta = \omega$. Therefore, soils A and B have porosities 0.44 and 0.52, respectively.

2. The total mass of a sample, M_t, is the sum of the mass of a constituent solid matrix M_s and water M_w, i.e., $M_t = M_s + M_w$. If the volume of a soil sample is V_t, this gives
$$\frac{M_t}{V_t} = \frac{M_s}{V_t} + \frac{M_w}{V_t}.$$
Since the bulk density (for a given saturation) is defined by $\rho_b = M_t/V_t$ and the solid density is $\rho_s = M_s/V_t$, we obtain
$$\rho_b = \rho_s + \frac{M_w}{V_t}.$$
Recalling that $M_w = \rho_w V_w$, where ρ_w is the density of water, yields an expression that you should use to compute the bulk density:
$$\rho_b = \rho_s + \rho_w \theta.$$

3. The water content at suction $\psi = 15cm$ is obtained by interpolating between the relevant values at the table.

Problem 3 (35 points)

Consider as a proposed hydraulic head distribution for a two-dimensional saturated system the following function:

$$h(x, y) = Ae^{-ax} \sin(ay + c) + B, \tag{1}$$

where a, c, A, and B are constants. Does h satisfy the appropriate differential equation?

Solution. Since flow is saturated and the medium is assumed to be homogeneous, the hydraulic head must satisfy the groundwater flow equation

$$S_s \frac{\partial h}{\partial t} = K \nabla^2 h.$$

Substitute h from (1) into this expression to verify whether it is a solution.