Quantification and Modeling of the Air Entrainment in the Stern Flow Region

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Motivation for studying the air entrainment by stationary 2-D and 3-D turbulent breakers formed at the stern of a surface ship

There are no detailed studies of three-dimensional stern breakers including the effect of the corner flow (stern geometry)

Our study:
- Two-dimensional deep-water breaker
  - air entrainment and bubble formation
- Three-dimensional deep-water breaker
  - Effect of the corner flow
Steady – unsteady measurements
Two-dimensional deep-water breaker
  – Effect of the Fr number on the air entrainment

Three-dimensional deep-water breaker
  – Mechanisms of air entrainment
  – Effect of the corner flow

Fresh and Sea Water
Experiments include using sluice gates as well as three-dimensional bodies resembling the stern of a surface ship.
Effect of the Boundary Layer on the Hull

Laminar/Turbulent
Mechanism of Air entrainment and Bubble Formation by a Two-Dimensional Deep-Water Stern Breaker
Unsteady Bubbly Flow Modeling
quasi-stationary toe

Top View. Stationary 2-D Stern Breaker
Passage time of the LS’s

• The passage time of the LS’s at a given location can be also measured.

\[ l(x_{\text{fix}}, t) = \int y \, \text{image}(x_{\text{fix}}, t) \, dy \]

\[ f \approx 3 \text{ Hz} \]
Velocity and size of the LS’s

• Experimental conditions: 
  \( U_0 = 2.25 \text{ m/s}; \Delta h = 8 \text{ cm} \)

• The LS’s move with a mean velocity \( U_{\text{mean}} = 0.45 \text{ m/s} \).

• The size \( l(x) \) of the LS’s growths almost linearly:
  \[ l(x) = \delta (x - x_0) \]
  \( \delta = 0.4 \left( \approx 20^\circ \right) \)
Unsteady Air entrainment/cavity collapse
Fresh water entrainment
Air entrainment length
Air entrainment model

$l = l(x)$
Results
Analysis of the mean velocity profiles. Self-similar horizontal velocity profiles.

- If the adimensional velocity profiles are plotted against \( \eta = (y - y_{0.1})/(y_{0.35} - y_{0.1}) \) they exhibit self-similarity.

- Close to the interface with the potential-flow region, the profiles depart from the profile proposed by Townsend (1976), \( U = (1 + \text{erf}((\eta - \eta_0)/C))/2 \).

Discussion
Growth rate of the mixing layer (III)

- In the classical mixing layer, the growth rate \( \frac{d\ell}{dx} \) is given by:
  \[
  \frac{U_0 + u_{\text{min}}}{U_0 - u_{\text{min}}} \frac{d\ell}{dx} = 2\beta = 0.028
  \]
  being \( \beta \) the entrainment parameter (Townsend, 1976).

- In the present flow, assuming a velocity profile \( U = (1 + \text{erf}((\eta - \eta_0)/C))/2 \), the growth rate is given by:
  \[
  \frac{d\ell}{dx} = 1.11 - \frac{d(y_{0.1} - y_{0.35})}{dx}
  \]

- The growth rates calculated for the three sets are:

<table>
<thead>
<tr>
<th>Set</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rate, ( \frac{d\ell}{dx} )</td>
<td>0.11</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>( \frac{(U_0 + u_{\text{min}})/(U_0 - u_{\text{min}})}{d\ell/dx} )</td>
<td>0.852</td>
<td>0.824</td>
<td>0.965</td>
</tr>
<tr>
<td>( \frac{(U_0 + u_{\text{min}})/(U_0 - u_{\text{min}})}{d\ell/dx} )</td>
<td>0.095</td>
<td>0.113</td>
<td>0.118</td>
</tr>
</tbody>
</table>
Air entrainment and bubble formation by a three-dimensional deep-water stern breaker

– Effect of the corner flow
Air Entrainment Mechanism

Steady 3-D Breaker

Top view

Side view
Unsteady Bubbly Flow Modeling

FF1052 stern wave
Bibliographical review

HYDRAULIC FLOWS IN SUDDEN EXPANSIONS:


EVOLUTION OF NONLINEAR WAVES ON A STEEP SLOPE:


CRITERIA BETWEEN SPILLING AND PLUNGING:


Experimental results
Different wave breaking regimes

Plunging

Spilling

Flow direction

Flow direction
Experimental results
Different wave breaking regimes

Plunging

Spilling

Flow direction

Flow direction
Flow configuration
**Vertex wave. Two configurations**

- Measurements of the free surface for two representative sets:

  **Plunging breaker**

  In this experiment, the crest separates at the red line.

  ![LIF visualization](image1)

  ![LIF visualization](image2)

  **Spilling breaker**

  No separation of the wave crest.

  ![LIF visualization](image3)

  ![LIF visualization](image4)
Which mechanism generates the wave?

- To investigate the formation of the wave, LDA measurements have been performed.
- Only two components (X and Z) could be measured. The third one was calculated by integration of the continuity equation:

\[
\nabla \cdot \vec{v} = 0 \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

- With the computed Y-component of the velocity, streamwise vorticity has been calculated for two cross-sectional sections: \(x=16\) mm (the closest to the plate that could be measured, magenta line in the figure) and \(x=40\) (where separation of the wave crest occurs, red line in the figure).
Which mechanism generates the wave?

There is a transversal “jet” approximately at the depth of the edge of the plate.

section where the wave crest separates
mechanism of wave generation

• Assuming gravity is the driving mechanism, the transversal velocity, \( v(z) \), would be of the order:

\[
v \sim \sqrt{2g (h_0 - h)}
\]

for \( h > h_1 \). In other words, the hydrostatic pressure driving the spilling is

\[
\Delta p \sim \rho g (h_0 - h)
\]

• For \( z < h_1 \), on the other hand, this hydrostatic pressure is opposed by another one of the order:

\[
\Delta p \sim \rho g (h_1 - h)
\]

• Therefore, the maximum \( v \) should be at \( z \approx h_1 \), as observed.

Y-velocity profiles (red lines) show the existence of the “jet”.
Transversal velocity/vorticity & free surface

Vorticity measured at X=16 mm (plunging wave)

Vy-Vz vector map @ X=16 mm

Streamwise vorticity @ X=16 mm
Modelling the wave formation

The streamwise velocity is uniform within ±20% in the region where the wave develops.
Modelling the wave formation

• Since the formation mechanism seems to be gravity driven, can it be described by some modification of the shallow water equations?

• Hager & Yasuda (1997) proposed a simplified model based on the shallow water equations to study a similar problem. The model assumes a velocity field \((u, v, w) = (U + u', v, w)\) with \(U \gg u', v, w\): 2D+Time approach.

• In our problem, the large aspect ratio of the wave guarantees \((u \gg v, w)\) (in fact, this has been corroborated experimentally), and one can assume \(U \gg u'\)

• Streamwise velocity contours have been measured at the sections indicated by red lines in the figure.
Wave generation mechanisms (I)

- Steady flow
- Velocity field can be written:
  \[ \vec{u} = \left( U_0 + u_x, u_y, u_z \right) \]
  with \( U_0 \gg (u_x, u_y, u_z) \)
- Gradients in the \( x \) coordinate are much smaller than in the \( y, z \)
  \[ \frac{\| \hat{u} \|_x}{\| \hat{u} \|_y, \| \hat{u} \|_z} \ll 1 \]

- Under these assumptions, there is an analogy between the evolution along the \( x \) axis of the wave profile in the \( y, z \) plane and the time evolution of a 2D wave: \( \frac{x}{U_0} \) emerges as a time-like variable
  \[ U_0 \frac{\| \hat{u} \|_x}{\| \hat{u} \|_t} \]
Wave generation mechanisms

- Assuming that the potential flow approximation is valid, the velocity field can be written as

\[ \vec{u} = \vec{N} f \]

- The velocity potential and the pressure are given by the following equations

\[ \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0 \]

\[ U_0 \frac{\partial f}{\partial x} + \frac{1}{2} \left( \frac{\partial f}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial f}{\partial z} \right)^2 + \frac{p}{\rho} + g z = 0 \]

- That must be completed with the boundary conditions

\[ U_0 \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial f}{\partial y} - \frac{\partial f}{\partial z} = 0 \quad \text{and} \quad p = 0 \quad \text{at the free surface,} \quad z = f(x, y) \]

\[ f \to 0 \quad \text{as} \quad z \to -\infty \quad \text{and} \quad f(x = 0, y) = f_0(y) \]
Modelling the wave formation

The model is clearly able to capture the mechanism of wave formation.

Numerical simulation. The purple line in the experimental measurements was used as initial condition.
Plunging-Spilling transition
Plunging-Spilling transition
**Plunging-Spilling transition**

- Once the phenomenon is known to be gravity-driven, a criterion is proposed to determine whether the wave will break in a plunging or spilling way.

- Since the maximum Y-velocity was found at the same depth as the vertex of the plate, the farther this point is from the minimum height of the free surface \((h_1)\), the easier it should be for the “jet” to separate and therefore, to form a plunging wave.

- On the other hand, the energy of the “jet” is given by the Froude number. So both parameters should be the relevant ones to characterize the wave.

- In fact, there exists a single curve in the space defined by both parameters that separates plunging from spilling waves.

- More measurements will extend this curve to the region of larger heights and lower Fr numbers, where currently there are no measurements.
If the height of the gate $h_0$ is kept constant, then

$$x_r = C U_0 \sqrt{2 \Delta h / g}$$

$$\Delta h = h_0 - h_1$$
Impingement velocity

• The position of the impingement point ("range" of the ballistic trajectory of the wave crest) has been measured for different sets, with velocities ranging from $U = 1$ to $3 \text{ m/s}$ and gate heights $h_o = 20$ to $80 \text{ mm}$.  

• A new Froude number may be defined based on the height difference: $\text{Fr}_g = \frac{U^2}{g(h_g - h_1)}$. Guided by the idea of a ballistic trajectory, we seek a relation of the form:

$$
\left( \frac{x_r}{(h_g - h_1)} \right)^2 \sim \text{Fr}_g
$$

• Indeed such a relation exists (red line), but it seems that the speed of the ballistic trajectory depends on an “excess” Froude number:

$$
\Delta \text{Fr} = \text{Fr}_g - \text{Fr}_g,\text{crit}
$$

with $\text{Fr}_{\text{crit}} \approx 20$. 
Experimental results

Breaking wave does not depend on the channel depth

Top view

Flow direction
Air Entrainment and Depth of Penetration
Penetration depth
**Detail of the entrainment region**

- The plunging wave impinges on the free surface, leading to the entrainment of large amounts of air at this point.

- The depth at which air bubbles are transported depends mainly on the velocity of the wave crest at the impingement position (Clanet & Lasheras, 1999).
\[ \frac{H}{D} = \frac{1}{2 \tan \alpha} \frac{V_0}{U_T}. \]

To estimate the mean velocity of the submerged jet as a function of the downstream location, we model the jet as an equivalent water jet with an initial momentum flux given by \( \pi \rho V_{j0}^2 D_{j0}^2 / 4 \), where \( V_{j0} \) and \( D_{j0} \) are the diameter and velocity of the jet at the point of impact with the flat interface.
\[ Mo = \frac{g \mu^2}{\rho_L \sigma^3} \]
FIG. 4. Non dimensional penetration depth \( H/D \cdot U_T/V_0 \) as a function of the penetration angle \( \theta \), for different diameters and velocities.

\[
\frac{H}{D} = \frac{(1 + \tan \alpha) \cos \theta + \tan \alpha \sin \theta \cos(\theta - \alpha)}{2 \tan \alpha} \frac{V_0}{\cos \alpha \frac{U_T}{}}. \quad (5)
\]
Air Entrainment Mechanism
Steady 3-D Breaker

Top view

Side view
Enhanced air entrainment mechanism (3-D)
Air entrainment mechanism due to the instability of the core of the corner vortex (3-D)

Unsteady traveling wave
Wavelength of the core instability of the “corner vortex”
Two-dimensional flow

- The region close to the toe of a bore has been studied experimentally using statistical correlation techniques.

- Accurate measurements of the advection velocities of the large scale eddies observed in the flow have shown that they move significantly slower than what would be expected in a single-phase mixing layer.

- The roller described in the literature emerges as an structure in the mean velocity field even though no recirculating motion exists at any particular time. The air entrainment at the toe is not the dominant mechanism.

- The shear layer grows linearly in a region that coincides fairly well with that occupied by the roller.

- Velocity profiles within this region exhibit a self-similar behavior. However, the self-similar velocity profile does not follow the one proposed by Townsend.

- The growth rate of the shear layer is about 3.5 larger than that expected for a classical mixing layer. This is consistent with the slower advection velocity of the large scale features.
Conclusions & Work in progress

3-D Corner Flow

- Entrainment due to the three-dimensional corner flow
  - Criterion for the transition
    - spilling breaker
    - plunging breaker

Air entrainment

- Entrainment depth
- Bubble size distribution

Future work:

- Extension to sea water
11.2 kts Elevation (Lidar sweeping away from transom)

Elevation (m) Time-series: Run 063 (06/08/2005), 11.2 kts; dx=0.1

Boat ---- data shown is sweeping aft....
Air entrainment and bubble formation by a two-dimensional deep-water stern breaker

– Effect of the Fr number on the air entrainment
Air entrainment and bubble formation by a three-dimensional deep-water stern breaker

- Mechanisms of air entrainment
- Effect of the corner flow
Experimental results
Plunging-Spilling Criterion

- Transition between both regimes seems to be determined by the flow steepness.
- Further experiments are needed

Both regimes alternate
Implementation of the model in URANS simulations

Since the LTF entrainment model is based on the behavior of the large flow structures, which cannot be simulated with URANS, the relevant information must be related to the mean value of the turbulent magnitudes of the flow.
Air entrainment model: entrainment length and turnover velocity

B) Entrainment length \( w \)
C) Turnover velocity \( u_l \)

Computed from PIV measurements for a large number of eddies
Self similar mean velocity profiles

- Mean velocity profiles collapse when properly dimensionalized.

\[
\frac{u(x, y) - u_{\text{min}}(x)}{U_0 - u_{\text{min}}(x)} = f\left(\frac{y}{d|x-x_0|}\right)
\]
Air entrainment model: growth rate

Measurements of the mixing layer growth:

- From the measured profiles the relation \( l = l(x) \) will be determined for different flow conditions.
- Work in progress for the determination of \( l_{\text{crit}} \).
deep-water stern breaker
3-D deep-water stern breaker

Mean Vertical Velocity
Upwards: Blue, Downward: Red
Air entrainment model (I): Modeling of a single LS

Entrainment of a single large turbulent feature:

- $l$: Characteristic size of the LS (assumed to be of the same order of its depth)
- $L \sim l$: Length of free surface perturbed by the LS
- $y$: Characteristic surface deformation
- $U_c$: Convective velocity of the LS
- $u_t$: Turnover velocity of the LS
- $\beta$: Maximum slope of the free surface
Air entrainment model (II): entrainment condition

The surface deformation can be estimated from a pressure balance

\[ \rho g y \gg \rho u_l^2 \Rightarrow y \gg \frac{u_l^2}{g} \]

Hypothesis: the entrainment occurs when the deformation of the surface, \( \beta \), is greater than a critical value \( \beta_{\text{crit}} \)

\[ \beta \gg \frac{y}{L} \mu \frac{y}{l} \gg \frac{u_l^2}{gl} \gg \beta_{\text{crit}} \]

\[ \text{equivalent to} \quad \text{Fr}_l = \frac{u_l^2}{gl} > \text{Fr}_{\text{crit}} \]
Air entrainment model (III): quantification

\( W = \) Entrainment length of the LS  \( u_e = \) Entrainment velocity

Assuming that \( \rho u_e^2 \) is proportional to a characteristic pressure difference \( \Delta p = \rho u_l^2 - Fr_{crit} \rho gl \), dimensional analysis yields:

\[
    u_e = \alpha \sqrt{gl} \sqrt{Fr_l - Fr_{crit}}
\]

For the entrainment length:

\[
    \frac{W}{l} = W \left( Fr_l \right)
\]

Thus the entrainment rate of a LS per unit span:

\[
    \frac{\dot{q}}{\sqrt{gl^3}} = \alpha W \left( Fr_l \right) \sqrt{Fr_l - Fr_{crit}}
\]
Model integration into a RANS code

A) Entrainment condition:

\[ Fr_l = \frac{u_l^2}{gl} > Fr_{crit} \]

To compute the local Froude number, \( Fr_l \), the following estimation of the parameters is proposed

\[
\begin{align*}
  u_l &= \frac{U_0 + U_S}{2} \\
  I &= \left( \frac{2}{U_0 + U_S} \right)^2 \int_{-\infty}^{h} (U_S + u)(U_0 - u) \, dy
\end{align*}
\]

B) Air entrainment per unit span:

\[ \dot{q}/\sqrt{gl^3} = \alpha W \left( Fr_l \right) \sqrt{Fr_l - Fr_{crit}} \]

In order to implement the model, the following constants must be measured from experiments (work in progress):

\[ Fr_{crit} \quad \alpha W \left( Fr_l \right) \]
Wave generation mechanisms

- Two mechanisms may explain the development of the wave:
  - Deep water analogous to the shallow water “roll waves” (Dressler 1949, Yu & Kevorkian 1992 and others)
  - Shear between the lower high-speed stream and the upper spilling flow + vortex stretching effect due to small variations in $u_x$
Transversal velocity/vorticity & free surface
Which mechanism generates the wave?

Vorticity of opposite sign is observed at the height of the plate.

This section is as close to the plate as we could measure.
Side View

2-D Stern Breaker

Deep-Water
Mean Horizontal Velocity showing the so called “roller” reported in previous studies (LDV measurements)
Impingement Velocity & Plunging-Spilling Transition

- The “range” of the ballistic trajectory depends on the ratio of the two dimensionless parameters that we used to determine the plunging/spilling criterion.

- The value found for the critical Froude number \( (F_{rg, \text{crit}} \approx 20) \) agrees fairly well with the value of the ratio \( Fr / (h_g - h_1) / (h_0 - h_1) \approx 24 \) measured at the transition condition.

- Transition should be: \( F_{rg} > F_{rg, \text{crit}} \approx 20 \).
Experimental results

Wave breaking regime: Plunging